

Bivariate drought frequency analysis using the copula method

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Abstract Droughts are major natural hazards with significant environmental and economic impacts. In this study, two-dimensional copulas were applied to the analysis of the meteorological drought characteristics of the Sharafkhaneh gauge station, located in the northwest of Iran. Two major drought characteristics, duration and severity, as defined by the standardized precipitation index, were abstracted from observed drought events. Since drought duration and severity exhibited a significant correlation and since they were modeled using different distributions, copulas were used to construct the joint distribution function of the drought characteristics. The parameter of copulas was estimated using the method of the Inference Function for Margins. Several copulas were tested in order to determine the best data fit. According to the error analysis and the tail dependence coefficient, the Galambos copula provided the best fit for the observed drought data. Some bivariate probabilistic properties of droughts, based on the derived copula-based joint distribution, were also investigated. These probabilistic properties can provide useful information for water resource planning and management.

1 Introduction

Drought is an insidious hazard of nature. Unlike many natural disasters that are sudden, droughts result when there is less than normal precipitation over an extended period of time. The departure from normality can produce serious agricultural, environmental, and socioeconomic damage. For example, during 1998–2001, ten out of 28 provinces in Iran were affected by one of the worst and most prolonged droughts in the country's history, leaving an estimated 37 million people (over half of the country's population) vulnerable to food and water shortages. Twenty provinces experienced precipitation shortfalls during the winter and spring of 2001 (Raziei et al. 2009).

Since droughts are strictly related to stochastic phenomena, such as precipitation or streamflow, the proper way to investigate droughts is by means of probability theory and stochastic process methods (Shiau 2006). The probabilistic characterization of droughts is extremely important, primarily in regions where accurate water resource planning and management requires a detailed knowledge of water shortages due to meteorological droughts (Rossi et al. 1992).

A significant amount of research has been performed on the probabilistic characterization of droughts (Yevjevich 1967; Fernández and Salas 1999; Shiau and Shen 2001; Cancelliere and Salas 2004). However, the majority of this work has dealt with univariate analysis while drought is a complex phenomenon that is characterized by a few randomly correlated variables. Therefore, a univariate analysis of drought characteristics cannot account for the significant correlation between variables.

A multivariate description provides a more complete characterization of droughts. However, in cases of drought, multivariate models are difficult to establish since different

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distribution functions are often used to fit various drought attributes (e.g., severity and duration). Thus, by applying traditional multivariate frequency analysis methods, it is not possible to precisely describe drought characteristics (Shiau 2006; Song and Singh 2010a, b, and references therein).

Over the last decade, copulas have emerged as a method for addressing multivariate problems in several disciplines (Salvadori et al. 2007). Copulas are a very powerful tool for quantifying the dependence structure between correlated quantities. Frequency analysis using a copula method, does not assume that variables are independent or normal or that they have the same type of marginal distributions (i.e., the marginal distributions of individual variables can be of any form and the variables can be correlated). Such characteristics are major advantages of this method (Zhang and Singh 2007 and references therein).

Detailed theoretical backgrounds and descriptions for the use of copulas can be found in Joe (1997), Nelsen (2006), and Salvadori et al. (2007). Copulas have been adopted for hydrological studies of multivariate flood frequency analysis (Favre et al. 2004; Zhang and Singh 2006; Shiau et al. 2006; Grimaldi and Serinaldi 2006a; Kao and Govindaraju 2007), for rainfall frequency analysis (De Michele and Salvadori 2003; Grimaldi and Serinaldi 2006b; Zhang and Singh 2007; Singh and Zhang 2007), for estimating groundwater parameters (Bardossy 2006), geostatistical interpolation (Bardossy and Li 2008), and for remote sensing data processing (Gebremichael and Krajewski 2007; AghaKouchak et al. 2010).

Using copulas to conduct bivariate frequency analysis of droughts, although still recent, has become more popular. Shiau (2006) constructed a joint drought duration and severity distribution using bivariate copulas. Copula-based joint probabilities and return periods for drought duration and severity satisfactorily fit the empirical values. Shiau et al. (2007) performed a bivariate assessment in order to investigate hydrological droughts for the Yellow River in northern China; and employed the Clayton copula with an exponential distribution for drought duration and a gamma distribution for drought severity. Shiau and Modarres (2009) used the Clayton copula for a bivariate analysis of drought severity and duration at the Abadan and Anzali gauge stations in Iran. Their results revealed that drought severity in a humid region may be more severe if high rainfall fluctuations exist within a region. Kao et al. (2009) used copulas to perform a spatio-temporal drought analysis for the Midwestern USA and adopted a copula-based joint deficit index (JDI) for describing overall drought status and compared it to the Palmer drought severity index. Their results indicated that the copula-based JDI provides information for drought identification and further allowed them to determine a month-by-month assessment for future drought recovery. Kao and Govindaraju (2010) indicated

that the JDI index was capable of accounting for the seasonality of precipitation and streamflow marginals. The JDI was also stated to be able to reflect both emerging and prolonged droughts in a timely manner and to allow a month-by-month drought assessment, such that the required amount of precipitation for achieving normal conditions in the future could be computed. Mirakbari et al. (2010) proposed a regional bivariate analysis for meteorological drought in the Khuzestan province, which is located in the southwest of Iran, and specified homogenous regions of drought characteristics (e.g., duration and severity) based on a L-moments analysis. The bivariate distribution of drought was then constructed using the copulas for each homogeneous region.

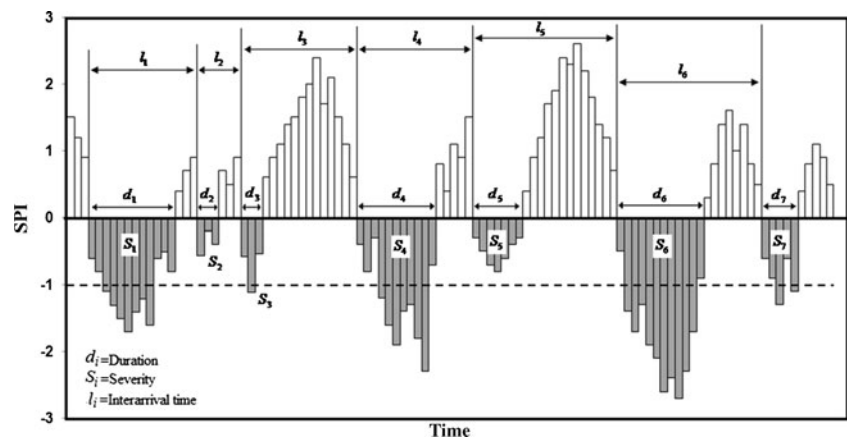
Recently, a few frequency analysis of drought have been performed using three- and four-dimensional copulas. However, the analyses were more complex mathematically relative to the bivariate case (e.g., Serinaldi et al. 2009; Wong et al. 2010; Song and Singh 2010a, b).

In the majority of previous work, especially in the case of drought analysis, the tail dependence has not been considered (with the exception of the work of Serinaldi et al. (2009)), even though the tail dependence test is an essential step that is related to modeling extremes (Poulin et al. 2007).

In this study, several families of two-dimensional copulas were applied in order to construct a joint drought duration and severity distribution using the monthly precipitation data obtained from the Sharafkhaneh rain gauge station. The best-fitted copula was selected based on an error analysis and tail dependence coefficient. Then, based on the derived copula-based joint distribution, some bivariate probabilistic properties of droughts, such as, the joint probabilities, bivariate return periods, conditional probabilities, and conditional return periods were investigated. Also, the new definition of multivariate return period proposed by Salvadori and De Michele (2010) is considered and compared with one presently used within the literature. These probabilistic properties provide useful information for drought assessments and can be used for water resources planning and management.

2 Definition of meteorological droughts

In this work, the standardized precipitation index (SPI) developed by McKee et al. (1993) was employed for defining drought events (Fig. 1). The SPI has advantages of statistical consistency, and the ability to describe both short-term and long-term drought impacts through various time scales of precipitation anomalies. In general, studies have indicated the usefulness of the SPI in quantifying different drought types (Edwards and McKee 1997; Hayes et al.

Fig. 1 The definitions of drought characteristics

1999). The probabilistic nature of the SPI allows it to be compared to various locations and variables, and it can be further interpreted in terms of recurrence intervals (or return periods). Therefore, the SPI has been widely adopted as a general tool for drought assessments (e.g., Hayes et al. 1999; Raziei et al. 2009). Also, most researchers have adopted the SPI for developing a copula-based joint drought deficit index (Shiau 2006; Shiau and Modarres 2009; Wong et al. 2010). Droughts are distinguished mainly by their duration and severity. Both of these characteristics play an important role in drought management (Dracup et al. 1980).

McKee et al. (1993) defined droughts as periods for which the SPI fell below -1 and subsequently increased above -1 . For their method, wet and dry conditions are classified according to the SPI values listed in Table 1. Some drought events may have prolonged drought duration but a moderate SPI for each period that does not reach -1.0 or less. The cumulative SPIs for such events (e.g., the 5th event in Fig. 1), however, are likely greater than those for events with a short duration that have a SPI less than -1.0 (e.g., the 3rd event in Fig. 1). Serious water supply and other drought-related problems are also caused by this type of event. As a result, according to Loukas and Vasilades (2004), a drought event is defined as a continuous period for which the SPI is below zero (Shiau 2006). Therefore,

Table 1 The weather classification by SPI values (McKee et al. 1993)

SPI value	Category
2.00 or more	Extremely wet
1.50 to 1.99	Severely wet
1.00 to 1.49	Moderately wet
-0.99 to 0.99	Near normal
-1.49 to -1.00	Moderately dry
-1.99 to -1.50	Severely dry
-2.00 or less	Extremely dry

drought duration is a continuous negative-SPI period while drought severity is defined as the cumulative values of the SPI within the drought duration, expressed using the following equation (McKee et al. 1993):

$$S = - \sum_{i=1}^D \text{SPI}_i \quad (1)$$

where D is drought duration, and S is drought severity.

Due to the existence of a correlation between drought characteristics, a univariate drought frequency analysis does not fully characterize drought potential. Bivariate frequency analysis can be used for considering the occurrence and effect of two drought characteristics simultaneously. Additionally, since drought duration and severity generally follow different distributions, joint multivariate models of droughts are difficult to establish. For this work, copulas were used to link fitted univariate distributions in order to construct a bivariate joint distribution.

3 Copulas

A copula is a multivariate distribution function that links joint probability distributions to their one-dimensional marginal distributions (Nelsen 2006). The theoretical basis of a copula was first introduced by Sklar (1959). For a bivariate case, according to Sklar's theorem, if two random variables x and y follow the arbitrary marginal distribution functions $F_X(x)$ and $F_Y(y)$, respectively, there then exists a copula, C , that combines these two marginals to give the joint distribution function, $F_{X,Y}(x, y)$, as follows (Nelsen 2006):

$$F_{X,Y}(x, y) = C(F_X(x), F_Y(y)) \quad (2)$$

Equation 2 indicates that a copula can describe a multivariate distribution in terms of a univariate distribution. If the marginal distributions $F_X(x)$ and $F_Y(y)$ are

continuous, the copula function C is unique, and the joint probability density function then becomes the following:

$$f_{X,Y}(x,y) = c(F_X(x), F_Y(y))f_X(x)f_Y(y) \quad (3)$$

where $f_X(x)$ and $f_Y(y)$ are the density functions corresponding to $F_X(x)$ and $F_Y(y)$, respectively, and c is the density function of C , which is defined as follows:

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \quad (4)$$

where u and v are the univariate cumulative distribution functions.

Many families of copulas exist and include the following: (1) Elliptical (normal and t); (2) Archimedean (Clayton, Gumbel, Frank, and Ali-Mikhail-Haq); (3) Extreme Value (Gumbel, Husler-Reiss, Galambos, Tawn, and t -EV); and (4) other families (Plackett and Farlie-Gumbel-Morgenstern). However, only a few of them have found practical applications (Joe 1997; Nelsen 2006; Capéraa et al. 2000; Fang et al. 2002; Abdous et al. 2004). Among the various families of copulas the Archimedean and the extreme value (EV) are more popular for hydrologic applications.

For an Archimedean copula, a generator φ exists such that the following relationship holds:

$$C(u,v) = \varphi^{-1}\{\varphi(u) + \varphi(v)\} \quad (5)$$

The generator φ is the continuous, strictly decreasing function, as defined in $[0, 1]$ and $\varphi(1)=0$. If $\varphi(1)=-\ln t$, then (u, v) are independent; in other words $C(u,v)=uv$.

Nelsen (2006) listed some important one-parameter families of Archimedean copulas, along with their generators, their range of the parameters, and some special and limiting cases.

In addition, a bivariate copula is considered to be an EV if a convex function exists, namely the Pickands dependence function, $A: [0, 1] \rightarrow [1/2, 1]$, which satisfies the constraints of $A(0)=A(1)=1$, and $\max\{t, 1-t\} \leq A(t) \leq 1$ for all $t \in [0, 1]$, such that (Nelsen 2006)

$$C(u,v) = \exp \left[\log(uv) A \left(\frac{\log v}{\log(uv)} \right) \right] \quad (6)$$

for all $(u,v) \in I^2$. In particular, if $A(t)=1$ then (u, v) are independent, and if $A(t)=\max\{t, 1-t\}$ then (u, v) are perfectly dependent (or comonotonic). Conversely, given a bivariate EV copula C , the corresponding Pickands dependence function (A) is given by the following:

$$A(t) = -\ln C(e^{-(1-t)}, e^t) \quad (7)$$

where $t \in [0, 1]$.

Salvadori et al. (2007) presented several environmental and geophysical examples by discussing univariate

and multivariate extreme value theory, the copula approach, and the relationships between these two research areas.

In this study, some, one-parameter copulas, including the Clayton, the Ali-Mikhail-Haq, the Farlie-Gumbel-Morgenstern, the Frank, the Galambos, the Gumbel-Hougaard, and the Plackett were selected for determining the joint probability distribution of correlated drought variables since they are easy to construct and since many researchers have used them in this area of hydrology and, in particular, in drought frequency analysis (Shiau 2006; Shiau et al. 2007; Wong et al. 2008; Kao et al. 2009; Shiau and Modarres 2009; Mirakbari et al. 2010; Wong et al. 2010).

3.1 Dependence analysis

The problem of measuring the amount of association, and/or the dependence between the variables involved, is a central issue when modeling multivariate extremes. The traditional measure of dependence, Pearson's ρ linear correlation coefficient, displays some drawbacks as a measure for bivariate distributions. For example, this coefficient only involves a linear kind of dependence and may not exist when heavy-tailed variables are present. Thus, it may not be suitable for describing extremes (Joe 1997).

To overcome the shortcomings of Pearson's ρ , some nonparametric measures such as Kendall's τ and Spearman's ρ rank correlation have been considered (Joe 1997; Nelsen 2006). These measures always exist and model several types of associations (Salvadori et al. 2007).

Kendall's τ is a rank correlation coefficient used to determine a measure of dependence between the random variables commonly associated with copulas. The population version of Kendall's τ is defined as the difference between the probability of concordance and the probability of discordance, as follows:

$$\tau = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] \quad (8)$$

where $P[(X_1 - X_2)(Y_1 - Y_2) > 0]$ and $P[(X_1 - X_2)(Y_1 - Y_2) < 0]$ represent the probability of concordance and the probability of discordance, respectively (Nelsen 2006). For two pairs (x_i, y_i) and (x_j, y_j) , they are concordant when $(x_i - x_j)(y_i - y_j) > 0$, and discordant when $(x_i - x_j)(y_i - y_j) < 0$.

For a random sample of n bivariate observations, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, the sample version of Kendall's τ can be estimated from the observations, as follows:

$$\tau = \left(\frac{n}{2} \right)^{-1} \sum_{1 \leq i < j \leq n} \text{sgn}[(x_i - x_j)(y_i - y_j)] \quad (9)$$

where $i, j = 1, 2, \dots, n$, and

$$\text{sgn}(\psi) = \begin{cases} 1 & \text{if } \psi > 0 \\ 0 & \text{if } \psi = 0 \\ -1 & \text{if } \psi < 0 \end{cases} \quad (10)$$

The choice of copulas depends on the range of the dependence levels that the copula can describe, and it can be determined by Kendall's τ . For example, the Gumbel-Hougaard copula can only be applied for positive dependence. The Ali-Mikhail-Haq and the Farlie-Gumbel-Morgenstern copulas are only suitable for $-0.1807 < \tau < 0.3333$ and $-2/9 \leq \tau \leq 2/9$, respectively, while the Clayton and the Frank copulas are suitable for both positive and negative dependencies (Nelsen 2006; Salvadori et al. 2007). These copula functions, as well as the range of their dependence parameter are shown in Table 2. The dependence parameter, θ , is used to measure the degree of association between u and v .

An important concept in the frequency analysis of extreme hydrological events such as droughts is the tail dependence, which describes the amount of dependence in the upper-right quadrant tail or lower-left quadrant tail (Frahm et al. 2005; Poulin et al. 2007).

Ignoring the tail dependencies amongst multiple hydrological variables may cause a high uncertainty in extreme quantile estimations which consequently leads to incorrect decisions for hydrological design (Xu et al. 2010). Poulin et al. (2007) showed the importance of taking into account the tail dependence in bivariate frequency analysis (based on copulas) for risk estimations. They also compared three nonparametric estimators of the tail dependence coefficient by simulations with seven families of copulas. Also, the notion of tail dependence may provide a useful indication for choosing a suitable family of copulas for modeling a given phenomenon (Poulin et al. 2007).

The concept of tail dependence was introduced by Sibuya (1960), and the formal definition was provided by Joe (1997) and Salvadori et al. (2007).

For the bivariate case, the upper (lower) tail dependence is indexed by the upper (lower) tail dependence coefficient $\lambda_u(\lambda_L)$, as follows:

$$\lambda_u = \lim_{t \rightarrow 1^-} P\{F_X(x) > t | F_Y(y) > t\} \quad (11)$$

$$\lambda_L = \lim_{t \rightarrow 0^+} P\{F_X(x) < t | F_Y(y) < t\} \quad (12)$$

where F_X and F_Y are the cumulative distributions of the random variables X and Y , and t is the fixed value of a standard uniform variable.

The bivariate distribution function is said to be upper (lower) tail dependent if $0 < \lambda_u \leq 1$ ($0 < \lambda_L \leq 1$), and upper tail independent if $\lambda_u = 0$ ($\lambda_L = 0$).

The tail dependence coefficient can also be defined via the notion of the copula as defined below:

$$\lambda_u = \lim_{t \rightarrow 1^-} \frac{1 - 2t + C(t, t)}{1 - t} \quad (13)$$

$$\lambda_L = \lim_{t \rightarrow 0^+} \frac{C(t, t)}{t} \quad (14)$$

Tail behavior solely depends on the type of copula and not on the choice of the marginal distribution. While a Gaussian copula does not have upper or lower tail dependence, a number of non-Gaussian copulas have lower tail dependency, upper tail dependency, or even both (AghaKouchak et al. 2010). The tested copulas allow a different configuration of tail dependence, and include the Clayton ($\lambda_L \neq 0$, $\lambda_u = 0$), the Ali-Mikhail-Haq ($\lambda_L = 0$, $\lambda_u = 0$), the Farlie-Gumbel-Morgenstern ($\lambda_L = 0$, $\lambda_u = 0$), the Frank ($\lambda_L = 0$, $\lambda_u = 0$), the Galambos ($\lambda_L = 0$, $\lambda_u \neq 0$), the Gumbel-Hougaard ($\lambda_L = 0$, $\lambda_u \neq 0$), and the Plackett ($\lambda_L = 0$, $\lambda_u = 0$) (see Nelsen 2006 for more details).

In this study, we focused only on the upper tail dependence since we were interested in evaluating the occurrence of contemporaneous extreme events. Also, for

Table 2 The one-parameter copula families and the range of their dependence parameters (Nelsen 2006)

Family	$C(u, v)$	Range of θ
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$\theta \geq 0$
Ali-Mikhail-Haq	$\frac{uv}{1 - \theta(1-u)(1-v)}$	$-1 \leq \theta \leq 1$
Farlie-Gumbel-Morgenstern	$uv[1 + \theta(1-u)(1-v)]$	$-1 \leq \theta \leq 1$
Frank	$-\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right]$	$\theta \neq 0$
Galambos	$uv \exp \left\{ \left[(-\ln u)^{-\theta} + (-\ln v)^{-\theta} \right]^{-\frac{1}{\theta}} \right\}$	$\theta \geq 0$
Gumbel-Hougaard	$\exp \left\{ - \left[(-\ln u)^{\theta} + (-\ln v)^{\theta} \right]^{\frac{1}{\theta}} \right\}$	$\theta \geq 1$
Plackett	$\frac{1}{2} \frac{1}{\theta - 1} \left\{ 1 + (\theta - 1)(u + v) - \left[(1 + (\theta - 1)(u + v))^2 - 4\theta(\theta - 1)uv \right]^{\frac{1}{2}} \right\}$	$\theta \geq 0$

this work, it was important to obtain an overall measure of dependence, which is well covered by Kendall's τ .

Unlike Kendall's τ , the nonparametric counterpart of λ_u is not straightforward. Frahm et al. (2005) described three nonparametric estimators for λ_u , including the LOG (Coles et al. 1999), the secant (SEC) (Joe 1997), and the Caperaa, Fougères, and Genest (CFG) estimators (Capéraa et al. 1997). The LOG and the SEC estimators require a threshold, whereas the CFG estimator does not need a threshold and was utilized in this study. As per the CFG estimator, the nonparametric upper tail dependence coefficient is given by the following:

$$\hat{\lambda}_u^{\text{CFG}} = 2 - 2 \exp \left\{ \frac{1}{n} \sum_{i=1}^n \log \left[\frac{\sqrt{\log \frac{1}{u_i} \log \frac{1}{v_i}}}{\log \frac{1}{\max(u_i, v_i)^2}} \right] \right\} \quad (15)$$

where $u_i = F_X(x_i)$ and $v_i = F_Y(y_i)$. Although the underlying assumption is that the copula can be approximated using an extreme value copula, Frahm et al. (2005) have shown that this assumption is not strong and that the estimator performs well even when the copula does not belong to the extreme value class (Serinaldi 2008). Furthermore, the relationship between Kendall's τ and λ_u can be exploited for the preliminary selection of a copula (Serinaldi 2008; Serinaldi et al. 2009).

Another important statistical concept related to copula-based analysis is Kendall's distribution function, which offers a cumulative probability measure for the region of $B_C(t) = \{(u, v) \in [0, 1]^2 | C(u, v) \leq t\}$, (Genest and Rivest 1993):

$$K_C(t) = P[C(u, v) \leq t] \quad (16)$$

where $t \in I=[0,1]$ is a probability level and $B_C(t)$ is the part of the unit square lying on, or below, and to the left of, the isoline of C having a level t . In the bivariate extreme value case, the K_C of the $B_C(t)$ is given by the following (Ghoudi et al. 1998):

$$K_C(t) = t - (1 - \tau_C)t \ln t \quad (17)$$

where τ_C is the value of the Kendall's τ associated with the copula C . Clearly, bivariate EV copulas with the same value of τ share the same function, K_C (Durante and Salvadori 2010).

The empirical estimator of Kendall's distribution function is given by the following (Genest et al. 2009):

$$K_n(t) = \frac{1}{n} \sum_{i=1}^n I(w_i \leq t), \quad t \in [0, 1] \quad (18)$$

where,

$$w_i = \frac{1}{n} \sum_{j=1}^n I(x_j < x_i, y_j < y_i) \quad (19)$$

and where $I(A)$ denotes the indicator variable of the logical expression A and takes the value 0 if A is false and 1 if A is true.

Kendall's distribution function can help project multivariate information onto a single axis. Therefore, it can be applied for illustrating a goodness of fit of copulas onto a single dimension (along t). Kendall's distribution function also plays a fundamental role in the new definition of a consistent multivariate return period (see Durante and Salvadori 2010; Salvadori and De Michele 2010 and references therein).

3.2 Fitting copulas to drought variables

When fitting copulas to meteorological drought variables of severity and duration, it is important to identify a suitable copula and to select best-fitted parameter. Parametric, semi-parametric, and nonparametric procedures are available for selecting the appropriate value of the dependence parameter in a copula-based model (Joe 1997; Genest and Rivest 1993; Genest et al. 1995).

In this study, a parametric estimation procedure known as the inference function for margins (IFM), as proposed by Joe (1997), was used to estimate the copula dependence parameter since it is a simple form and since it is commonly used in hydrology (Shiau 2006; Shiau et al. 2007; Shiau and Modarres 2009). The IFM method includes two procedures, as follows: (1) marginal distributions are computed from the observed values, and (2) the copula dependence parameter, θ , is estimated through the maximization of the log-likelihood function of copula:

$$\ln L(d, s; \lambda, \alpha, \beta, \theta) = \ln L_C(F_D(d), F_S(s); \theta) + \ln L_D(d; \lambda) + \ln L_S(s; \alpha, \beta) \quad (20)$$

where $\ln L_C$ is the log-likelihood function of the copulas. By substituting the estimated parameters of $\hat{\lambda}$, $\hat{\alpha}$, and $\hat{\beta}$, the log-likelihood function ($\ln L$) can be maximized in order to obtain the copula parameter $\hat{\theta}$.

Once an appropriate copula is identified, the dependence structure can be modeled. The procedure is conceptually similar to the selection of suitable PDFs for marginal distributions.

4 Application

4.1 Data

In this study, monthly precipitation data from October 1967 to September 2007 from the Sharafkhaneh station was selected in order to explore its drought characteristics. The Sharafkhaneh station is located in the East

Azərbaycan Province, in the northwest of Iran, next to Urmia Lake (at a latitude of $38^{\circ}11' \text{ N}$, a longitude of $45^{\circ}28' \text{ E}$, and an elevation of 1,252 m). As reported by the West Azarbaijan Environment Organization, due to drought and increased demands for agricultural water in the lake's basin, the level of Urmia Lake decreased by approximately 6 m during 2000–2010. The salinity of the lake has risen to more than 300 g/L during recent years, and large areas of the lake bed have been desiccated (Eimanifar and Mohebbi 2007). The location of Sharafkhaneh station is shown in Fig. 2. All of the required data were obtained from the Iranian Water Resources Institute and the Iranian Meteorological Organization.

Mean annual precipitation for the Sharafkhaneh station is 254 mm, and approximately 59% of annual precipitation is received during the period from February to June. An apparent seasonal distribution can be observed in the historical precipitation data from this station.

Monthly SPIs from 1967 to 2007 from the Sharafkhaneh gauge station were used to determine drought characteristics. Monthly precipitation was computed based on the aggregated daily values. After data processing (including quality assurance with neighboring stations), the monthly SPI was calculated, as demonstrated in Fig. 3 for this 40-year period. Then, the drought durations and according severities were abstracted from 84 observed droughts for the period 1967–2007 (Fig. 4).

A strong correlation was observed between drought duration and drought severity. The correlation coefficient between drought duration and drought severity was 0.9151. The average duration and the average severity for the observed droughts were 2.33 months and 1.57, respectively. The two longest drought durations, of 6 months, both occurred from July to December of 1973 and 1998. These drought events also had the greatest drought severity, 4.23 and 5.45, respectively.

4.2 Univariate CDFs for drought severity and duration

Drought severity and duration are commonly fitted with gamma and exponential distributions, respectively (e.g., Mathier et al. 1992; Shiau and Shen 2001; Shiau et al. 2007; Shiau and Modarres 2009). The probability density function of the gamma distribution is expressed as follows:

$$f_S(s) = \frac{s^{a-1}}{\beta^a \Gamma(a)} e^{-s/\beta}, \quad s > 0 \quad (21)$$

where s is the drought severity; a and β are the shape and the scale parameters, respectively; and Γ is the gamma function. The probability density function of the exponential distribution is as follows:

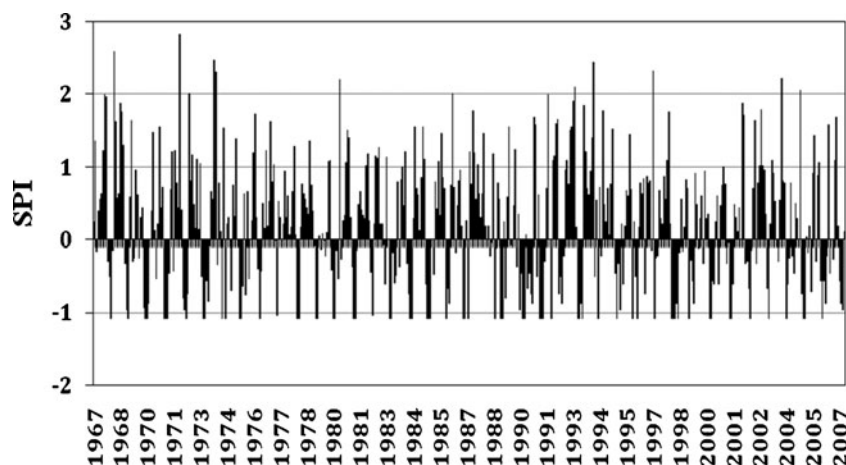
$$f_D(d) = \frac{1}{\lambda} e^{-d/\lambda}, \quad d > 0 \quad (22)$$

where d is the drought duration and λ is a parameter. The a , β , and λ should be estimated from the observed data.

Fig. 2 The location of the Sharafkhaneh rainfall gauge station



Fig. 3 The monthly SPI of the Sharafkhaneh gauge station (1967–2007)



The parameters of gamma and the exponential distributions were estimated using the maximum likelihood method, which resulted in $\hat{\alpha} = 1.190$, $\hat{\beta} = 2.289$, and $\hat{\lambda} = 2.333$. The obtained cumulative drought duration and severity distribution functions are as follows:

$$F_D(d) = 1 - e^{-d/2.333} \quad (23)$$

$$F_S(s) = \int_0^s \frac{t^{0.190}}{2.289^{1.190} \Gamma(1.190)} e^{-t/2.289} dt \quad (24)$$

Both models for duration and severity are presented, respectively, in Figs. 5 and 6, and show good agreement with the observed drought data.

4.3 Goodness-of-fit test for marginal distributions

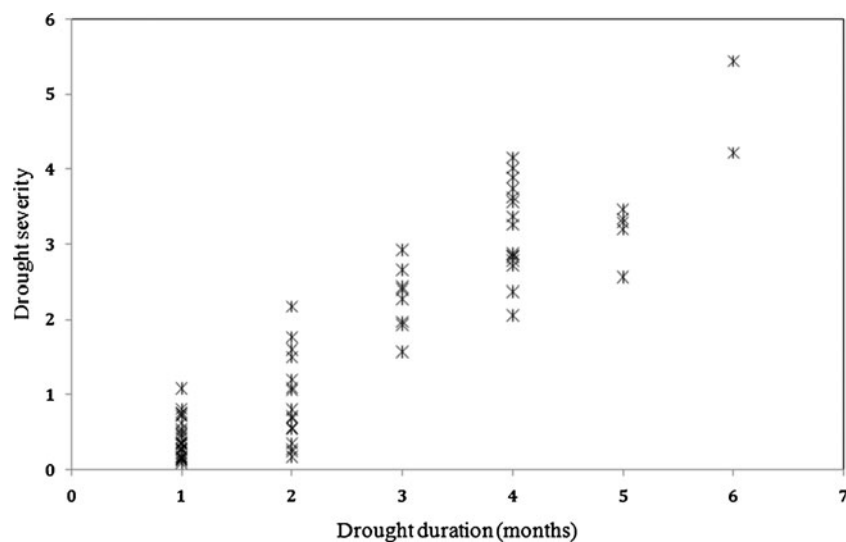
The Kolmogorov–Smirnov goodness-of-fit test was used to detect whether or not the proposed models could be used to

represent the observed data. The critical value for a sample size of 84 is 0.133, at the 10% significance level. The maximum deviations between the observed data and the proposed models of drought duration and severity for Sharafkhaneh were 0.06 and 0.12, respectively, which both were less than the critical values. The results indicate that the hypothesis for the proposed gamma and exponential distributions for modeling drought severity and duration at Sharafkhaneh could not be rejected and that the fitted models were quite acceptable.

Since drought duration and severity followed different distributions, and since a high correlation existed between them, copulas were employed to construct the joint distribution.

At first, seven families of copulas, including the Clayton, the Ali-Mikhail-Haq, the Farlie-Gumbel-Morgenstern, the Frank, the Galambos, the Gumbel-Hougaard and the Plackett, were selected in order to assess their fitness to the observed drought data. However, the calculated Kendall's τ between the severity and duration of drought using the elongated time series obtained from the Sharafkhaneh

Fig. 4 A scatter plot of the observed drought duration and the drought severity for the Sharafkhaneh gauge station



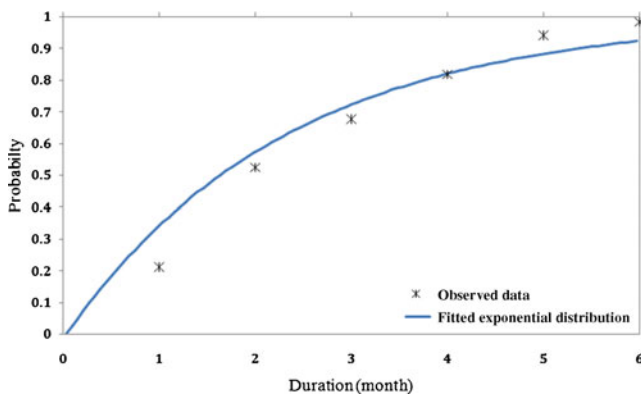


Fig. 5 A comparison of the observed drought duration and the fitted exponential distribution

rain gauge was found to be 0.604. Since the obtained Kendall's τ value was outside of the domain of the Ali-Mikhail-Haq and the Farlie-Gumbel-Morgenstern copulas, these copulas were automatically excluded from further analysis.

Finally, the five remaining copulas, including the Clayton, the Frank, the Galambos, the Gumbel-Hougaard, and the Plackett, were fitted to the drought variables. As shown in Table 3, the dependence parameters for each copula function were estimated using the IFM method (Eq. 20).

4.4 Goodness-of-fit test for bivariate distribution

For this study, the best-fitted copula was specified in two ways. For the first specification, the nonparametric empirical copula was computed and compared with the values of the parametric copulas. The parametric copula that was closest to the empirical copula was defined as the most appropriate choice (Genest and Favre 2007). The second specification was based on the comparison of parametric

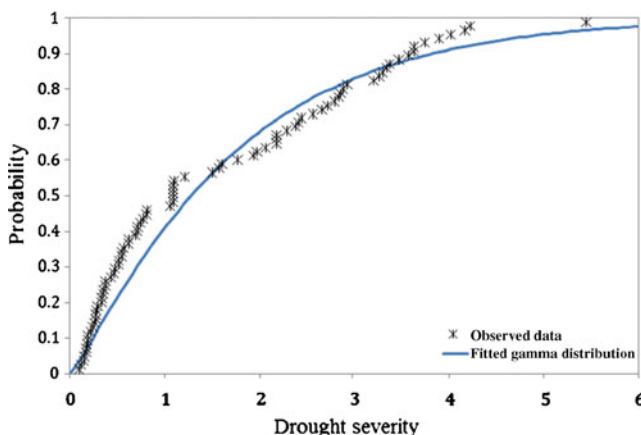


Fig. 6 A comparison of observed drought severity and the fitted gamma distribution

Table 3 The values of a parameter, the log-likelihood function, the root mean square error (RMSE), the Akaike information criterion (AIC), and the Nash–Sutcliffe equation (NSE) for copula distributions

Copula	Parameter	Log-likelihood function	RMSE	AIC	NSE
Clayton	1.381	−260.305	0.0811	522.609	0.9049
Frank	7.894	−240.307	0.0438	482.613	0.9723
Galambos	1.967	−232.79	0.0418	467.580	0.9748
Gumbel-Hougaard	2.652	−233.119	0.0421	468.238	0.9743
Plackett	16.795	−242.084	0.0489	486.168	0.9653

and nonparametric values of the upper tail dependence coefficients (Ghosh 2010).

Empirical copulas are rank-based, empirically joint cumulative probability measures (Nelsen 2006). For the bivariate case, the empirical copula of the observed data (u_i, v_i) is as follows:

$$C_e(u_i, v_i) = \frac{1}{n} \sum_{i=1}^n I\left(\frac{D_i}{n+1} \leq u_i, \frac{S_i}{n+1} \leq v_i\right) \quad (25)$$

where n is the sample size; $I(A)$ denotes the indicator variable of the logical expression A and assumes a value of 0 if A is false and 1 if A is true; and the ranks of the i th observed duration and the severity data are represented as D_i and S_i , respectively. The measure of fit is based on how close the points are to the diagonal line.

In order to evaluate the fitted copula, the empirical copula was evaluated using the observed data, $C_e(u_i, v_i)$, and was compared with the fitted parametric copula that was evaluated using the observed data, $C_p(u_i, v_i)$.

The root mean square error (RMSE), the Akaike information criterion (AIC) (Akaike 1974), and the Nash–Sutcliffe equation (NSE) (Nash and Sutcliffe 1970) were used in order to select the best-fitted copula function. RMSE, AIC, and NSE can be expressed as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n [C_p(i) - C_e(i)]^2} \quad (26)$$

$$\text{AIC} = -2 \ln \text{ML} + 2k \quad (27)$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^n (C_p(i) - C_e(i))^2}{\sum_{i=1}^n (C_e(i) - \bar{C_e})^2} \quad (28)$$

where n is the sample size; C_p denotes the computed values of the parametric copula; C_e denotes the observed values of the probability obtained from the empirical copula; $\bar{C_e}$ is

the mean of the observed values of the probability obtained from the empirical copula; lnML denotes the maximized log-likelihood function; and k is the number of independently adjusted parameters. The model is more efficient when RMSE and AIC are close to zero, or when the NSE is close to one.

As shown in Table 3, the RMSE, the AIC, and the NSE for the considered copulas were compared. Based on the comparison it was determined that the Galambos and the Gumbel-Hougaard copulas had minimum RMSE and AIC, and maximum NSE among the all of the considered copulas. However, the Galambos copula provided the best fit, and a minor difference was observed between the goodness-of-fit measures of the Galambos and the Gumbel-Hougaard copulas. Therefore, both of these copulas can be chosen as potential candidates for constructing the bivariate drought distribution. However, in this study, the Galambos copula was selected to model the dependence structure between drought duration and drought severity.

Figure 7 shows a theoretical estimation obtained using the IFM procedure of the Galambos copula plotted versus the empirical copula for the Sharafkhaneh station. The relationship is close to the 45° line, and confirms the use of the Galambos copula for characterizing the dependence structure and for constructing the bivariate model.

As mentioned previously, tail dependence coefficients can be used in order to select a suitable copula. Parametric estimators of the upper tail dependence coefficient for the copulas considered are presented in Table 4, along with their values. Also, the upper tail dependence coefficient for drought severity and duration for the Sharafkhaneh station, obtained using the CFG estimator (Eq. 15), was 0.708, which when compared to Table 4, indicates that the Galambos was the best-fitted copula. However, minor differences existed between the upper tail dependence coefficients of the Galambos and the Gumbel-Hougaard copulas. The differences are similar to previously obtained

Table 4 Parametric estimators and values of the upper tail dependence coefficient for different copula distributions

Copula	λ_u	Value
Clayton	0	0
Frank	0	0
Galambos	$2^{-1/\theta}$	0.703
Gumbel-Hougaard	$2-2^{1/\theta}$	0.701
Plackett	0	0

results for the comparison of the empirical and the parametric copulas.

Therefore, Galambos-based bivariate drought distributions for the Sharafkhaneh station can be expressed as follows:

$$F_{D,S}(d,s) = F_D(d)F_S(s) \exp \left[\left(-\ln F_D(d) \right)^{-1.967} + \left(-\ln F_S(s) \right)^{-1.967} \right]^{-\frac{1}{1.967}} \quad (29)$$

Since various drought duration and severity combinations can result in the same probability, joint probabilities are demonstrated as contour lines. Figure 8 illustrates the contours of the joint probability for drought duration and the severity using the Galambos copula, using observed drought data from the Sharafkhaneh gauge station.

4.5 Joint probability of droughts

Joint probabilities of droughts characteristics are important for drought management. The probability that both the drought duration and the severity simultaneously exceed certain thresholds is useful information for environmental

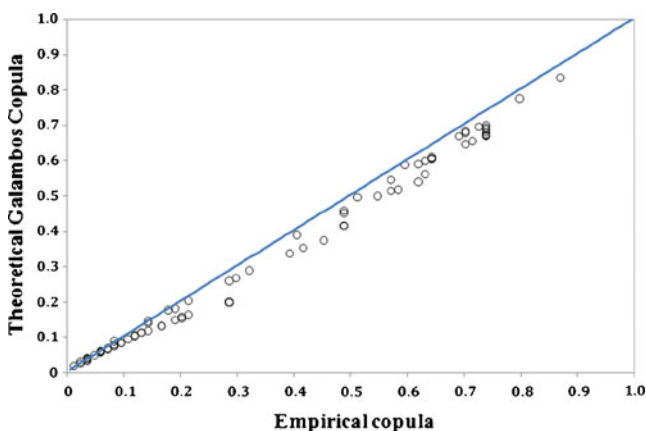


Fig. 7 The fitted Galambos copula versus the Empirical copula for the Sharafkhaneh gauge station. The solid line indicates the 45° line

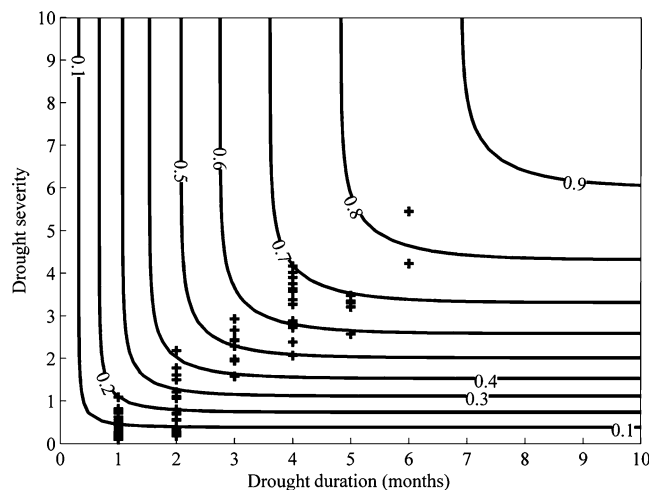


Fig. 8 Contours for the joint probability of drought duration and drought severity using observed drought data obtained from the Sharafkhaneh gauge station

and government agencies for improving water systems management under drought conditions. This probability cannot be obtained using separate drought duration and severity analysis. However, the probability can be easily derived in terms of copulas (Shiau 2006), using the following relationship:

$$P(D \geq d, S \geq s) = 1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s)) \quad (30)$$

Also, conditional drought distributions can be easily derived from the copula-based bivariate drought distribution. It is useful for evaluating the drought severity distribution given that the drought duration exceeds a certain threshold d' (Eq. 31) or the drought duration distribution given that drought severity exceeds a certain threshold s' (Eq. 32). These conditional probabilities of duration and severity can be obtained using the following derived copula according to Shiau (2006):

$$P(S \leq s | D \geq d') = \frac{F_S(s) - C(F_D(d'), F_S(s))}{1 - F_D(d')} \quad (31)$$

$$P(D \leq d | S \geq s') = \frac{F_D(d) - C(F_D(d), F_S(s'))}{1 - F_S(s')} \quad (32)$$

Figures 9 and 10 show the conditional drought severity distribution given that the drought duration exceeds various thresholds, as well as the conditional drought duration distributions, given that the drought severity exceeds various thresholds. Both figures imply that the conditional drought severity distribution and the conditional drought duration distribution decrease with drought duration and drought severity, respectively.

The graphs are very handy for determining the probability of drought severity for the given drought duration exceeding a certain value (Fig. 9), and, conversely, the probability of drought duration for the given drought

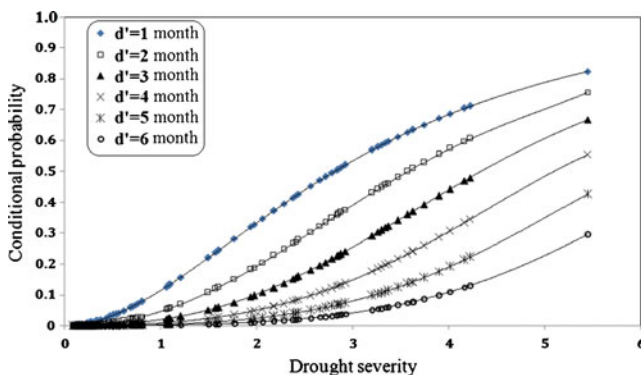


Fig. 9 The conditional distribution of drought severity given drought duration exceeding a certain value, d'

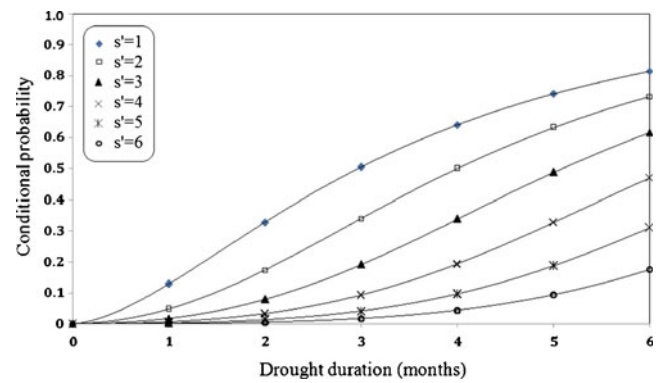


Fig. 10 The conditional distribution of drought duration given drought severity exceeding a certain value, s'

severity exceeding a certain value (Fig. 10). For example, according to Fig. 9, the probabilities for drought severities less than 2 and 4, given a drought duration that exceeds 3 months, are, respectively, equal to 0.101 and 0.441. Probabilities for drought durations less than 2 and 4 months, given a drought severity that exceeds 2, are equal to 0.175 and 0.501, respectively (Fig. 10).

4.6 Joint drought return period

The planning and management of water resources systems under drought conditions requires the estimation of the return periods of drought events, which are characterized by high severities (Kim et al. 2003). Shiau and Shen (2001) defined the return period for droughts with duration greater than or equal to a certain value, and the return period for drought severity greater than or equal to a certain value (Eqs. 33 and 34)—as a function of the expected drought inter-arrival time and, the cumulative drought duration, and the drought severity distribution functions, respectively, as follows:

$$T_D = \frac{E(L)}{1 - F_D(d)} \quad (33)$$

$$T_S = \frac{E(L)}{1 - F_S(s)} \quad (34)$$

where T_D is the return period with a drought duration greater than or equal to a certain value; T_S is the return period with a drought severity greater than or equal to a certain value; L is the inter-arrival time that is defined as the period between the beginning of a drought and the beginning of the next drought; and $E(L)$ is the expected drought inter-arrival time (Fig. 1).

The average drought inter-arrival time, as estimated from the observed data, was 0.9875 months. The drought duration and the drought severity obtained for 2-, 5-, 10-,

20-, 50-, and 100-year return periods are summarized in Table 5. For example, for the longest and most severe observed drought event (July to December of 1998), with duration of 6 months and a severity of 5.45, the return periods, as defined separately by drought duration and drought severity, are 7.3 and 7.8 years, respectively.

Since both severity and duration play an important role in drought assessment and management, it would be interesting to estimate the joint return period of drought characteristics.

According to Shiau (2006), the joint return period of drought duration (D) and severity (S) is defined for two cases: the return period for $D \geq d$ and $S \geq s$, and the return period for $D \geq d$ or $S \geq s$, which is denoted by T_{DS} and T'_{DS} , respectively, as follows:

$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \quad (35)$$

$$T'_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \quad (36)$$

For example, the joint return period for $D \geq 2$ and $S \geq 4$ is 4.6 years, and the joint return period for $D \geq 2$ or $S \geq 4$ is 1.9 years.

Similar to the joint probability, the joint return period for various drought duration and severity combinations can be demonstrated using contour lines. Figures 11 and 12 illustrate the contours of the joint drought duration and severity return period for ($D \geq d$ and $S \geq s$), and for ($D \geq d$ or $S \geq s$) respectively at the Sharafkhaneh station.

Adapting the results from the contour plots for a given return period, there are different combinations of severity and duration. Although it is generally believed that the return period should be adapted to the worst case scenario in terms of historical events. Salvadori and De Michele

Table 5 The return periods as defined separately by the drought duration and the severity

Return periods (years)	Drought duration (months)	Drought severity
2	2.12	2.04
5	4.87	4.35
10	6.95	6.04
20	9.02	7.71
50	11.77	9.89
100	13.85	11.54

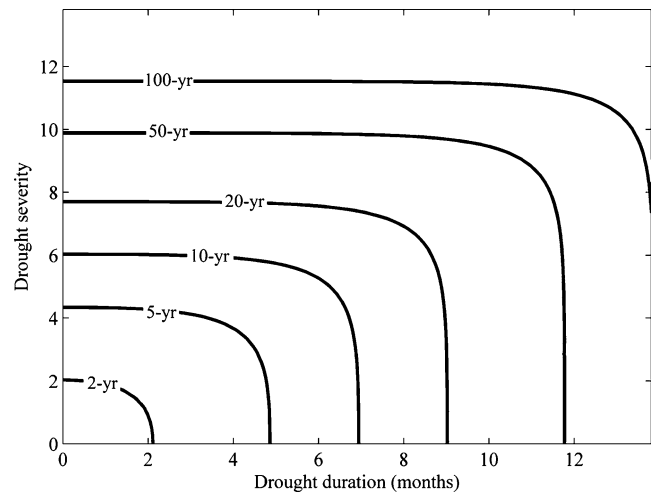


Fig. 11 The joint drought duration and severity return period, T_{DS} ($D \geq d$ and $S \geq s$)

(2010) found that using the standard definition of return period (Eq. 36) can result in underestimates of the correct value, and proposed the use of Kendall's return period, based on Kendall's distribution function as follows:

$$T_p = \frac{E(L)}{1 - K_C(t)} = \frac{E(L)}{1 - P\{u \in I^d : C(u) \leq t\}} \quad (37)$$

where $t \in I$ is the critical threshold that given by

$$t = \inf\{s \in I : K_C(s) = p\} = K_C^{[-1]}(p) \quad (38)$$

Clearly, since K_C is a distribution function, t represents the corresponding proper quantile of the order p . Salvadori et al. (2011) introduced coherent notions of multivariate threshold (critical layer) and total order in multi-dimensional Euclidean spaces for the calculation of Kendall's return period. They also presented a consistent

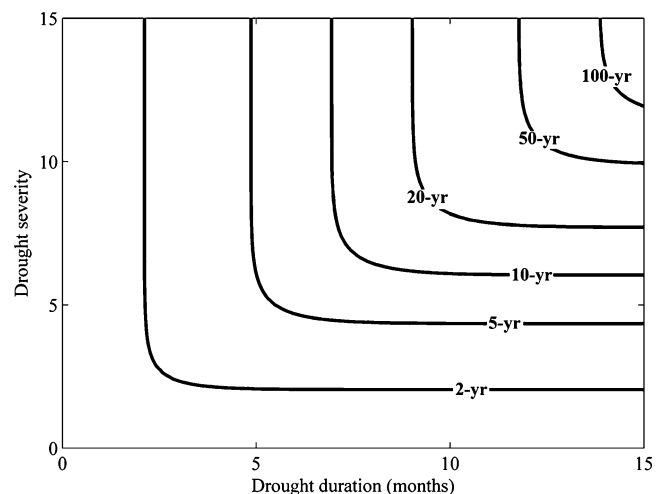


Fig. 12 The joint drought duration and severity return period, T'_{DS} ($D \geq d$ or $S \geq s$)

notion of multivariate quantile and introduced several approaches for the identification of critical design events when several dependent variables are involved.

Understanding the definition for Kendall's return period is practical. Suppose that a critical return period, T_p , is specified via design requirements. For example, water resources managers may be interested in designing a water-supply system that can provide sufficient water under a specified extreme drought event that (on average) occurs once every T_p years. Then, by inverting Eq. 37, a critical probability level, t , can be calculated, and a region, $B_C(t)$, of sub-critical, non-threatening events can then be identified.

Figure 13 displays Kendall's return periods (Eq. 37), as well as the standard return periods (Eq. 36) associated with all critical probability levels, $t \in I$. Note that, as a result of the limited sample size, estimates for the largest empirical return periods are spoiled.

For the sake of comparison, we also provide the corresponding standard return periods, as prescribed by Eq. 36. As indicated, the difference between the Kendall's return periods (correct) and the standard return periods (wrong) is considerable, especially, at $t > 0.65$. For example, according to Fig. 13, the standard return period that corresponds to the critical probability level of $t = 0.8$ is equal to 5 years; however, the corresponding Kendall's return period is approximately 16.5 years. Furthermore, it can be seen that the difference between Kendall's return periods and the standard return periods increases with an increase in t (Fig. 13).

As illustrated and discussed in Durante and Salvadori (2010) and Salvadori et al. (2011), Kendall's return periods (Eq. 37) are much larger than the standard return periods (Eq. 36) generally found in the literature. Clearly, the underestimates provided by the standard approach (i.e., a return period much smaller than the correct return period) causes an error in estimations of risk. Using Kendall's

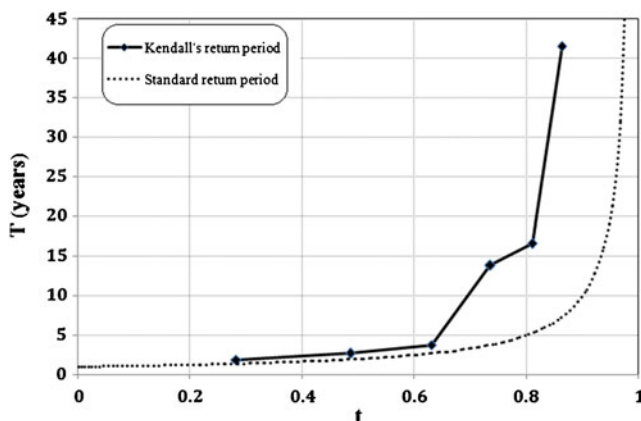


Fig. 13 Kendall's return period (T_p) based on the empirical Kendall's distribution function and the standard return period (T'_{DS}) based on the Galambos copula for the Sharafkhaneh gauge station

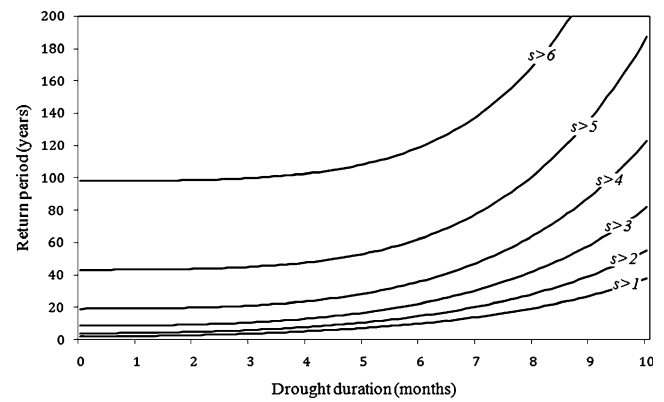


Fig. 14 The conditional return period of drought duration given that the severity is greater than a certain value, s

approach, a correct risk analysis can be performed (Salvadori and De Michele 2010).

According to Shiau (2006), the conditional drought duration and severity return period can also be defined for the following two cases: the return period of drought duration given a drought severity that exceeds a certain threshold, and the return period of drought severity given a drought duration that exceeds a certain threshold. Both conditional return period definitions for copula-based drought events are described below,

$$T_{D|S \geq s} = \frac{T_S}{P(D \geq d, S \geq s)} = \frac{E(L)}{[1 - F_S(s)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]} \quad (39)$$

$$T_{S|D \geq d} = \frac{T_D}{P(D \geq d, S \geq s)} = \frac{E(L)}{[1 - F_D(d)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]} \quad (40)$$

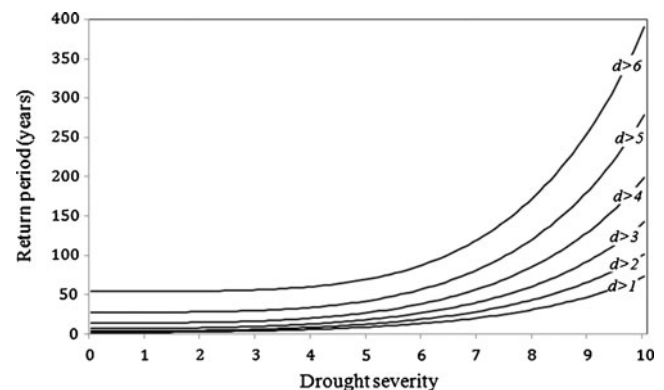


Fig. 15 The conditional return period of drought severity given that the duration is greater than a certain value, d

where $T_{D|S>s}$ denotes the conditional return period for D given $S>s$; and $T_{S|D>d}$ denotes the conditional return period for S given $D>d$.

Joint and conditional return periods of drought duration and severity are used by water resources managers and engineers as a hydraulic design criterion and provide useful information for evaluating risk (Shiau 2006; Song and Singh 2010a, b). For example, if a specific water-supply system cannot provide sufficient water under a situation for which drought severity exceeds a level of 3, given a drought duration that exceeds 2 months, according to Eq. 40, the return period for such a situation is 6.16 years. Figs. 14 and 15 demonstrate conditional return periods of drought duration and severity as, respectively, defined by Eqs. 39 and 40.

5 Conclusions

An assessment of meteorological drought for the Sharafkhaneh station in the northwest of Iran is provided. Drought, as defined by the SPI, is continuously below 0. Paired drought duration and severity data were abstracted from observed drought events. The exponential and the gamma distributions fitted well for drought duration and severity, respectively. A dependence assessment, based on Pearson's classical coefficient and Kendall's τ , indicated a high dependence between duration and severity. Thus, a bivariate distribution for drought duration and drought severity was considered.

Since drought duration and drought severity were fitted using different distributions, commonly used bivariate distributions, such as the bivariate exponential and the bivariate gamma distributions, could not be applied. Therefore, copulas were employed in order to link the fitted univariate models and construct the bivariate distribution for drought duration and severity. Copula parameters were estimated using the method of IFM. Several types of copulas were tested in order to select the best-fit copula. Based on the error analysis and the tail dependence test, the Galambos copula provided the best fit to observed drought data. The bivariate model of drought characteristics (e.g., the joint probabilities, as well as the joint return periods of the combination of drought severity and duration, as well as the conditional probability and the conditional return period of drought characteristics) was investigated for the Sharafkhaneh station.

Also, Kendall's return period was considered and compared with the standard definition of return period. In general, we observed that at a particular critical probability level, t , Kendall's return periods are much larger than standard return periods, and that this difference increases with an increase in t .

The longest and the most severe drought event observed for the Sharafkhaneh station occurred during July to

December of 1998, with duration of 6 months and with a severity of 5.45. Separate return periods, as defined by drought duration and drought severity for this drought event, were 7.3 and 7.8 years, respectively. The joint return period for a drought duration exceeding 6 months and a drought severity exceeding 5.45 was $T_{DS}=10.4$ years. The joint return period for a drought duration exceeding 6 months or a drought severity exceeding 5.45 was $T'_{DS}=5.9$ years. The Kendall's return period for this situation was $T_p=27.7$ years. However, if minor non-drought events interrupting this drought were ignored, the longest and the most severe drought event for the Sharafkhaneh station occurred during April 1990 to February of 1991, with the cumulative drought duration and drought severity of 11 months and 6.8, respectively. Joint return periods for this drought event were $T_{DS}=40.8$ and $T'_{DS}=13.5$ years, respectively. The Kendall's return period for this event was $T_p=92.4$ years.

According to the definition of the conditional return period, a return period for drought duration exceeding 6 months, given drought severity exceeding 5.45 as obtained from Eq. 39, is 82.7 years. The return period for a drought severity exceeding 5.45, given a drought duration exceeding 6 months as obtained from Eq. 40, is 76.9 years. The joint probability for a drought duration exceeding 6 months and a drought severity exceeding 5.45, as derived from Eq. 30, is equal to 0.1. The probability for a drought severity less than 5.45, given a drought duration exceeding 6 months as obtained from Eq. 31, is equal to 0.3; and the probability for a drought duration less than 6 months, given drought severity exceeding 5.45 as obtained from Eq. 32, is equal to 0.24.

Since the Sharafkhaneh station is located beside Urmia Lake, our results can be used to assess the risk of drought on Urmia Lake water decline. The results presented here indicate that the long and severe drought occurrence probability at the Sharafkhaneh station is high, and that drought events are a frequent phenomenon in this region. Therefore, in order to prevent water level decline, salinity increase, and the subsequent drying of Urmia Lake, accurate water resources management of the lake's basin is necessary.

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