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Analysis of meteorological drought in northwest Iran using the Joint Deficit Index

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SUMMARY

Probabilistic assessment and prediction of drought provides valuable information for water resources planners and policy makers for developing drought mitigation strategies. In this study an evaluation of drought conditions in northwest of Iran was performed by means of the Joint Deficit Index (JDI). Monthly precipitation data from 1970 to 2007 based on 50 gauge stations uniformly distributed across the area were used for calculating the JDI. Results show that the JDI provides a comprehensive assessment of droughts and that it is capable of reflecting both emerging and prolonged droughts reported in the data. Furthermore, the method provides a basis for determining the amount of precipitation required to reach normal conditions in future months (1–3 months examined in this study), and the exceedance probability of this precipitation amount. Performance evaluation based on 6 years of independent precipitation data from the region showed Critical Success Index of 0.61 (0.64) for the 1-month (3-month) ahead prediction of the drought conditions. The analysis in this study indicated a good skill in predicting the evolution of drought conditions for the region based on JDI evaluated from monthly precipitation data.

1. Introduction

Drought is a climatic extreme affecting more people than any other form of natural disaster (e.g. Wilhite, 2000). Of all natural disasters, droughts are globally occurring most frequently, have longer durations, impact larger areas and the most vulnerable populations on earth affecting food security through significant losses in agricultural production (e.g. WMO, 1997). Drought may occur in any part of the world, but duration and intensity of droughts vary greatly across different climatic zones (e.g. Wilhite, 1993). Drought causes serious agricultural, environmental and socioeconomic damages. For example, the average annual cost in agriculture losses due to droughts in the United States is estimated between $6 and $8 billion. Another example is the 2001 drought in Iran caused by precipitation shortfalls in the winter–spring seasons that constitutes one of the worst and most prolonged droughts of the country's history. This drought affected about 37 million people (half of the country's population) and caused damages to agriculture and livestock estimated to about $2.5 billion (Agrawala et al., 2001).

When a drought event occurs, moisture deficits are observed in many hydrologic variables, such as precipitation, streamflow, soil moisture, snow pack, groundwater levels and reservoir storage. Droughts are categorized according to various types of deficits. For example, meteorological droughts are based on deficits in precipitation, agricultural droughts on deficits in soil moisture, and hydrologic droughts on streamflow deficits (Dracup et al., 1980). Assessment and prediction of drought provides valuable information for water resources planners and policy makers to cope with drought consequences. Because of the complex relationship among the different physical factors affecting the initiation and persistence of a drought, it is difficult to provide a precise definition of drought that would work in all circumstances. This is a main reason as to why policy makers and water resource planners have difficulty recognizing and planning for a drought. Therefore, drought management relies on statistical indices to decide when to start implementing water conservation or mitigation measures (Khadr et al., 2009).

Drought indices summarize different data on rainfall, snowpack, streamflow, and other water supply indicators into a comprehensive picture of drought occurrence (Heim, 2002). Several indices have been proposed by researchers for quantifying drought severity that are derived from hydro-meteorological variables. Some of the popular drought indices include: the Palmer Drought

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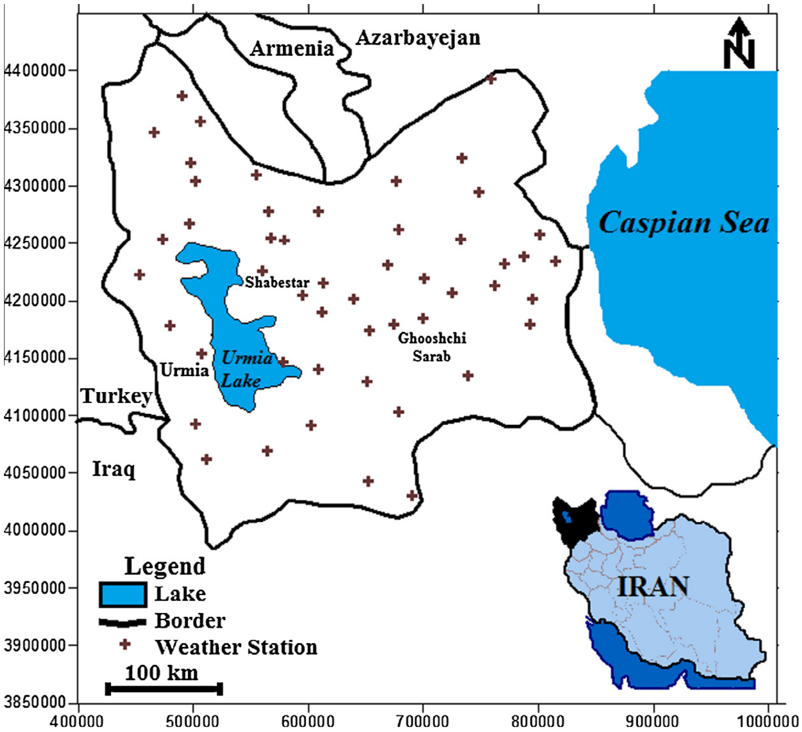

Fig. 1. Study area map and location of meteorological stations over northwest of Iran.

Severity Index (PDSI; Palmer, 1965), Crop Moisture Index (CMI; Palmer, 1968), Surface Water Supply Index (SWSI; Shafer and Dezman, 1982), Standardized Precipitation Index (SPI; McKee et al., 1993), the Reclamation Drought Index (RDI; Weghorst, 1996), Effective Drought Index (EDI; Byun and Wilhite, 1999), Streamflow Drought Index (SDI; Nalbantis and Tsakiris, 2009), Standardized Hydrological Index (SHI; Sharma and Panu, 2010), Standardized Precipitation Evapotranspiration Index (SPEI; Vicente-Serrano et al., 2010) Evaporative Drought Index (EDI; Yao et al., 2010), Regional Drought Area Index (RDAI; Fleig et al., 2011) and Agricultural Reference Index for Drought (ARID; Woli et al., 2012). Every index has its own strengths and weaknesses. Mishra and Singh (2010) have given a comprehensive review of the different drought indices summarizing their usefulness and limitations. Besides, a

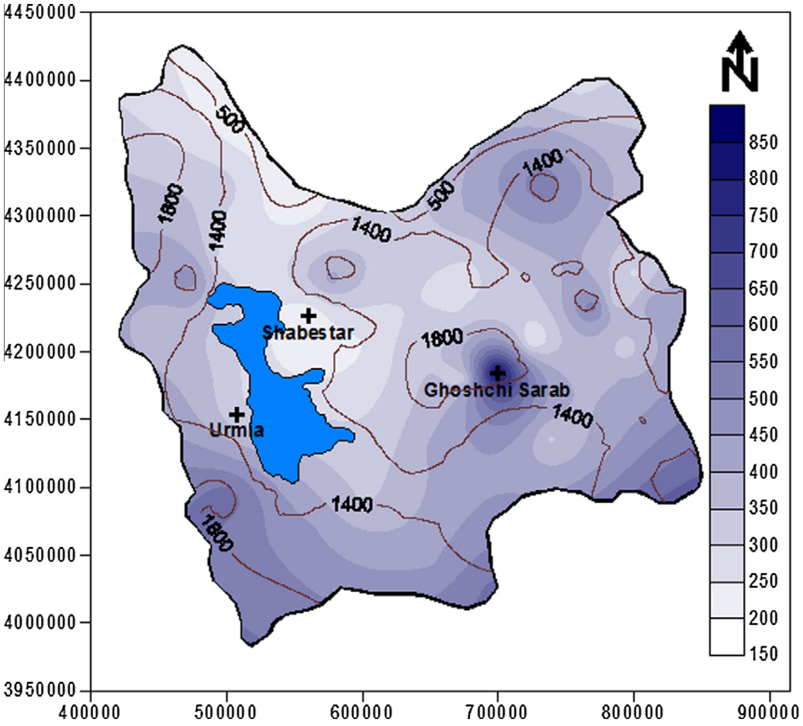

Fig. 2. Mean annual precipitation (mm) over northwest of Iran (1970–2007). The solid lines represent topographic contours.

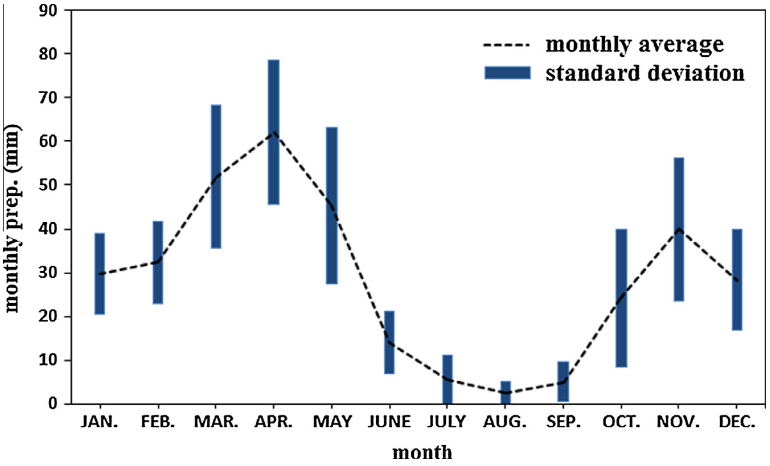

Fig. 3. The monthly average precipitation along with the corresponding standard deviation for Urmia station.

good review on remote sensing based drought indices is given in Bayarjargal et al. (2006) and Niemeyer (2008).

Among all drought indices, the Standardized Precipitation Index, SPI, is the most popular and is now widely used throughout the world for drought analysis (e.g. Hayes et al., 1999; Raziei et al., 2009; Ibrahim et al., 2010). SPI exhibits advantages in terms of statistical consistency, and the ability to describe both short-term and long-term drought impacts through the different time scales of precipitation anomalies. The probabilistic nature of SPI allows it to be comparable among various locations. Nevertheless, it can lead to confusion because inconsistent results may emerge under different time scales. Droughts have varying durations and SPI with pre-specified window size cannot capture this phenomenon adequately: emerging droughts cannot be depicted in long-term SPI values, while a prolonged drought may be interrupted by a short-term SPI value. Therefore, multiple SPIs with various temporal scales (e.g. 3-, 6-, 9-, 12-months) need to be examined in order to provide an overall assessment of a drought (Kao and Govindaraju, 2010). Besides, SPI cannot account for seasonal variability, i.e. a given amount of precipitation has different effects on moisture status depending on when it occurs. This is due to defining SPI based on the overall mean instead of monthly means. Therefore, Kao and Govindaraju (2010) proposed the modified SPI, hereafter named SPI^{bmod}, which is based on monthly means. Although, SPI^{bmod} can account for seasonal variability in precipitation data, multiple SPI_w^{mod} values at different temporal scales (e.g. w = 3-, 6-, 9-, 12-months) are needed to achieve an overall assessment of a drought.

To resolve the above issues, Kao and Govindaraju (2010) proposed a new index named Joint Deficit Index (JDI), which consists of joint distributions of multiple SPIs using copula functions. More details on calculating the JDI will be given in the Section 3. They showed that the JDI index is capable to account for seasonality of precipitation and streamflow marginal distributions. The index is able to reflect both emerging and prolonged droughts in a timely manner and also allows for a month-by-month drought assessment; such as determining the required amount of precipitation for reaching normal conditions in subsequent months.

The copulas used in JDI are multivariate distribution functions linking joint probability distributions to their one-dimensional marginal distributions (Nelsen, 2006). Copulas have been used in many hydrological applications including flood frequency analysis (Favre et al., 2004; Zhang and Singh, 2006; Shiau et al., 2006; Kao and Govindaraju, 2007), rainfall frequency analysis (De Michele and Salvadori, 2003; Zhang and Singh, 2007; Grimaldi and Serinaldi, 2006), estimating groundwater parameters (Bardossy, 2006), processing remote sensing data (Gebremichael and Krajewski, 2007; AghaKouchak et al., 2010) and the analysis of droughts (Shiau, 2006; Shiau et al., 2007). Specific to droughts, Shiau (2006) constructed a joint drought duration and severity distribution using the bivariate copulas. Shiau et al. (2007) did a bivariate assessment to investigate the hydrological droughts of the Yellow River in northern China. They employed the Clayton copula using the exponential distribution for drought duration and the gamma distribution for drought severity. Kao et al. (2009) have used copulas to perform a spatio-temporal drought analysis for the Midwestern US. They adopted a copula-based JDI to describe the overall drought status and compared it to the Palmer drought severity index. The results showed that the copula-based JDI provides information for drought identification, and further allows a month-by-month assessment for future drought recovery. Ibrahim et al. (2010) have applied bivariate copula for evaluating drought events of ten rain stations in Peninsular Malaysia, and showed that for the majority of stations the Galambos distribution provides the best fit. Recently, frequency analyses of droughts have been performed using three-dimensional and four-dimensional copulas that are more complex mathematically as compared to the bivariate case (e.g., Serinaldi et al., 2009; Song and Singh, 2010; Wong et al., 2010).

A few studies on bivariate drought analysis using copulas have been conducted in Iran; which is the focus area of this study. Specifically, Shiau and Modarres (2009) used the Clayton copula to do the bivariate analysis of drought severity and duration for Abadan and Anzali gauge stations in Iran. Results showed that drought in humid regions can be more severe if high rainfall fluctuations existed in that region. Mirakbari et al. (2010) proposed a regional bivariate analysis for meteorological droughts in Khuzestan province, south-west Iran. They specified homogenous regions of drought characteristics (e.g., duration and severity) based on L-moments analysis. Then, the bivariate distribution of drought was constructed using copulas for each homogeneous region. Recently, Mirabbasi et al. (2012) conducted a bivariate analysis of drought duration and severity using copulas for Sharafkhaneh station, north-west of Iran. They investigated some bivariate probabilistic properties of droughts, based on the derived copula-based joint distribution. The results indicated that the long and severe drought occurrence probability at the Sharafkhaneh station is high, and that drought events are a frequent phenomenon in this region.

This study aims to perform an evaluation of drought conditions in northwest of Iran using the JDI approach using copulas, and develop a method for predicting the wetness conditions based on exceedance probability thresholds determined from climatological

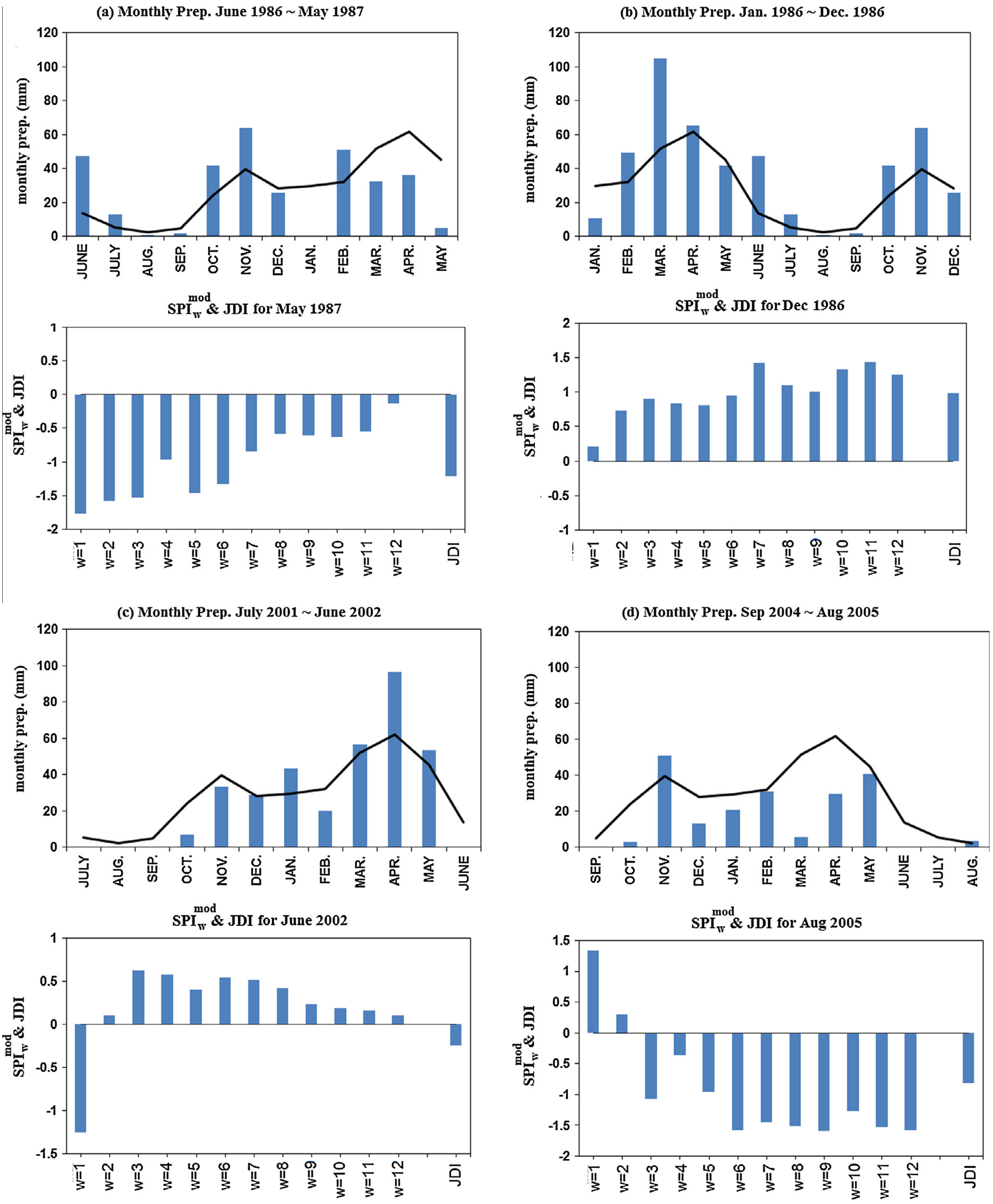

Fig. 4. Examples of JDI, SPI_w^{mod}, $w = 1,\ldots,12$, and the corresponding 12-month precipitation values for Urmia station. The solid line shows the long-term monthly mean values.

data. The performance of the proposed method will be evaluated using independent verification data from 6 years of precipitation observations from the region. In the next section we discuss the study region and data. In Section 3 we present the JDI implementation and in Section 4 we describe the results from implementing the JDI method on 32 years of precipitation data from the region, and provide an independent verification for drought severity prediction based on the last 6-year record of precipitation data. We

Table 1
Contingency table of SPI_1^{mod}, SPI_6^{mod}, SPI_{12}^{mod} and JDI for emerging drought events.

Predicted	Observed			
	Yes		No	
	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI
Yes	2827 1382	1594 2873	0 937	533 196
No	1372 2667	2584 1329	3468 2681	2956 3269

Table 2
Contingency table of SPI_1^{mod}, SPI_6^{mod}, SPI_{12}^{mod} and JDI for prolonged drought events.

Predicted	Observed			
	Yes		No	
	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI
Yes	205 276	276 286	0 73	70 8
No	71 0	0 44	73 0	3 11

close this paper with conclusions and recommendations for further research.

2. Data and study area

Thirty-eight years (January 1970 to December 2007) of daily precipitation data from 50 meteorological stations in the north-west of Iran is used in this study to analyze drought events. As shown in Fig. 1 the selected stations represent a good spatial coverage across the region. Daily precipitation records were first processed in terms of data gaps using neighboring stations to estimate missing precipitation values and then converted to monthly precipitation. The study domain covers an area of approximately 100,500 km^2. The mean annual precipitation in the area ranges significantly (Fig. 2), namely, from 198 mm (at the Shabestar station) to 845 mm (at the Ghoshchi Sarab station). As shown in Fig. 2 most parts of the study area receive less than 400 mm precipitation per year. The greatest amount of precipitation depth is observed over the mountainous areas and particularly the eastern region that receives more rainfall due to the Caspian Sea effect. The temporal variability is also significant. Fig. 3 shows the mean monthly precipitation, and the corresponding standard deviation, for the Urmia station. As noted from the figure, about 56% of the annual precipitation is received during the period of February to May, which exhibits a strong seasonal distribution. The standard deviation is also high, ranging between 50% (in wet months) and 100% (in the dry months) of the mean, which indicates significant inter-annual variability of the monthly precipitation. Similar patterns on seasonal and inter-annual variations are observed in the other station measurements from the region. The above data record was divided into two periods for the purpose of this study: (1) 1970–2001 for the evaluation of JDI copulas, which is discussed in Section 3, and (2) 2002–2007 for independent verification of drought predictions using the JDI approach, which is discussed in Section 4.

Table 3
CSI, POD and FAR of SPI_1^{mod}, SPI_6^{mod}, SPI_{12}^{mod} and JDI for emerging and prolonged drought events.

	Emerging events		Prolonged events	
Skill score	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI	SPI_1^{mod} SPI_{12}^{mod}	SPI_6^{mod} JDI
CSI	0.673 0.277	0.338 0.653	0.743 0.791	0.798 0.846
POD	0.673 0.341	0.382 0.684	0.743 1.000	1.000 0.867
FAR	0.000 0.404	0.251 0.064	0.000 0.209	0.202 0.027

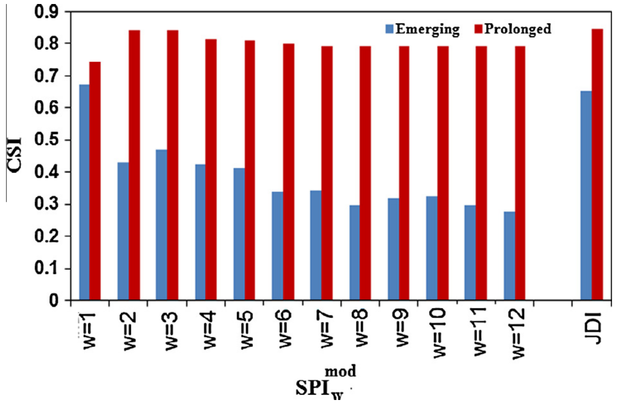

Fig. 5. CSI values of SPI_w^{mod} and JDI for emerging and prolonged drought events.

3. Joint Deficit Index

The Joint Deficit Index (JDI) used in this study was developed by Kao and Govindaraju (2010). To calculate JDI, the joint distribution of multiple modified SPIs associated with various time scales (SPI_w^{mod}, $w = 1,\ldots,12$) is constructed through copulas. According to Sklar's theorem (Sklar, 1959), if we have d random variables (x_i, $i = 1,\ldots,d$) with univariate marginal distributions (F_i), and joint distribution function H, then there exists a d-copula function (C) that combines these univariate marginal distributions to give the joint distribution function (H) as:

$$H_{X_1,\ldots,X_d}(x_1,\ldots,x_d) = C(F_1(x_1),\ldots,F_d(x_d)) = C(u_1,\ldots,u_d) \quad (1)$$

The detailed theoretical background and descriptions for the use of copulas can be found in Joe (1997), Nelsen (2006) and Salvadori et al. (2007).

To calculate the modified SPI, first we determined the m-month precipitation, X_w, grouped by its ending month to form X_w^{month} (month = January, February,…, December). Namely, the series $X_w(t)$ are subdivided into 12 sub-series:

$$X_w^{month}(g) = X_w(12(g-1)+m) = X_w(t) \quad (2)$$

where g is the year index, $m = 1$ (January), 2 (February),…, 12 (December) is the month index, and t is the time index, $t = 12(g-1)+m$. For example, $X_1^{January}$ represents January precipitation, and X_5^{August} represents the 5-month precipitation accumulation from April to August. By fitting distributions separately for each group (namely, $u_w^{January} = F_{X_w^{January}}(x_w^{January}), u_w^{February} = F_{X_w^{February}}(x_w^{February}),\ldots,u_w^{December} = F_{X_w^{December}}(x_w^{December})$, $w = 1,2,\ldots,12$), the modified SPI can be computed by taking the inverse normal random variables from cumulative probability values (u_w^{month}):

$$SPI_w^{\text{mod}} = \phi^{-1}(u_w^{month}) = \phi^{-1}\left(F_{X_w^{month}}(x_w^{month})\right) \quad (3)$$

Therefore, we can determine 12 modified SPI (SPI_w^{mod}) values for every month relating to different time integration windows ($w = 1,2,\ldots,12$); thus, the modified SPI has 12 times more cases than the conventional SPI to account for seasonality.

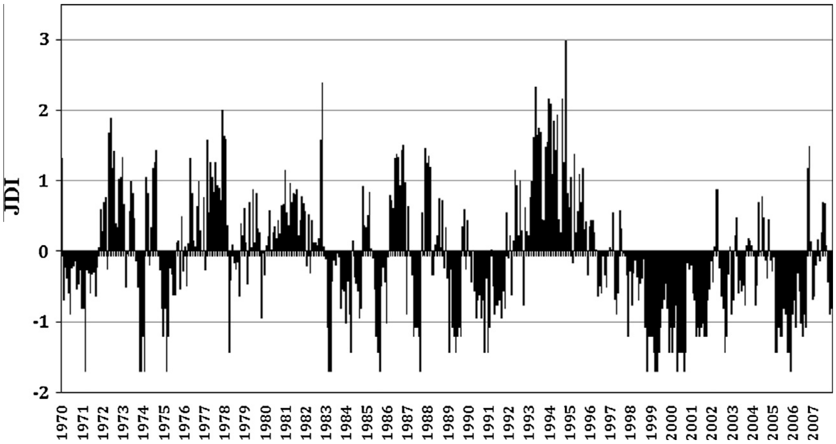

Fig. 6. Monthly JDI determined based on Urmia station data (1970–2007).

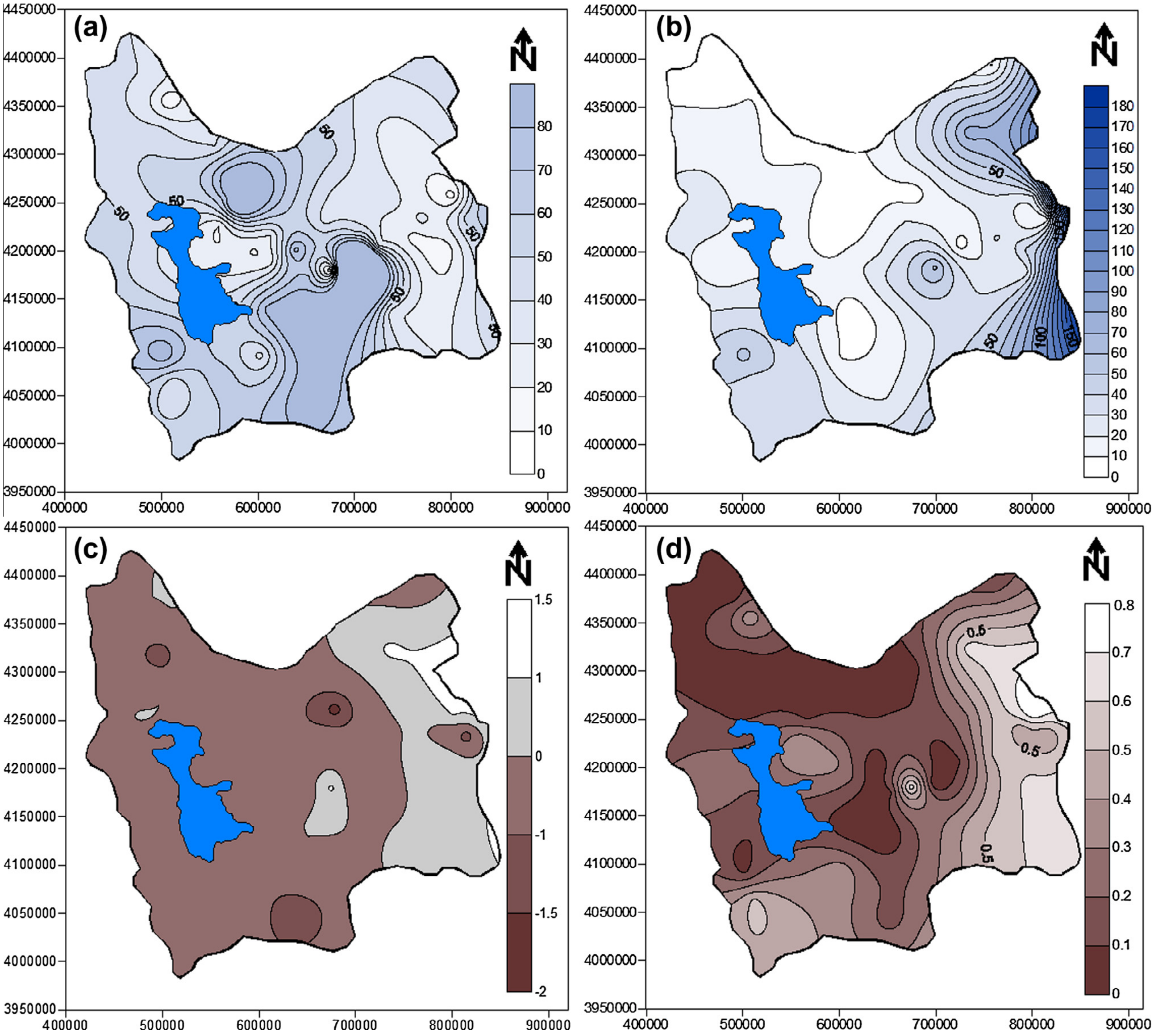

Fig. 7. Regional illustration of (a) required precipitation (mm), (b) observed precipitation (mm), (c) JDI, and (d) exceedance probability of the required precipitation to achieve normal status (JDI = 0) for northwest of Iran (January 2001).

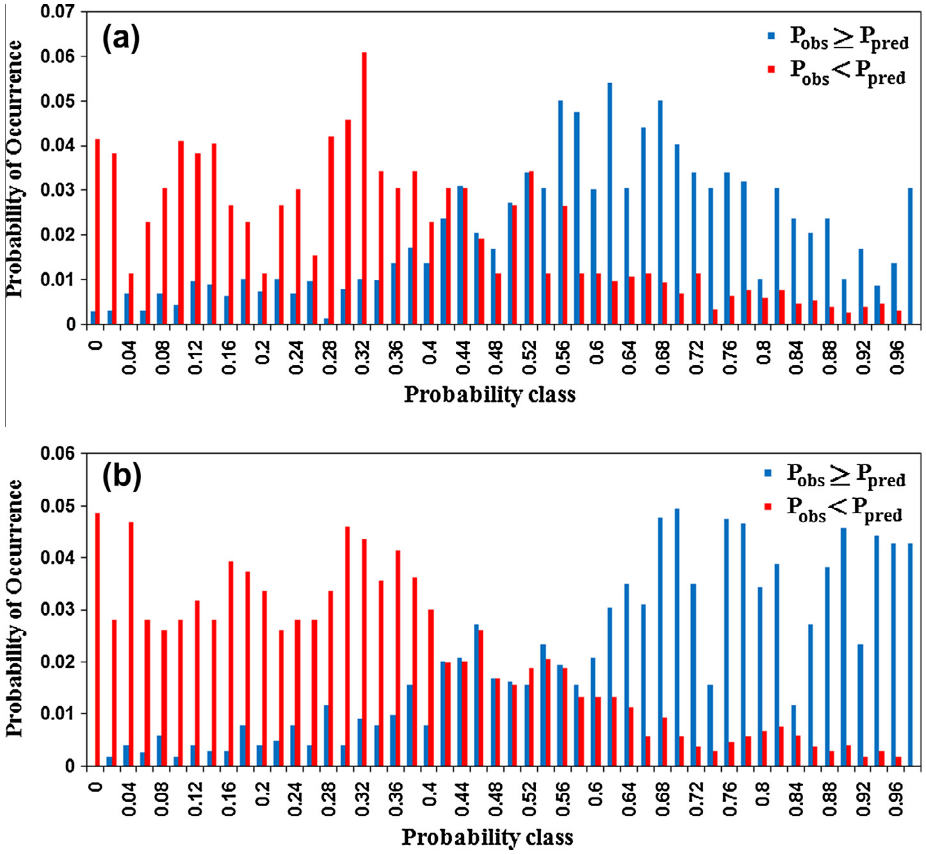

Fig. 8. Frequency histograms of the predicted exceedance probabilities when the 1-month-ahead observed precipitation is greater or equal (blue bars) or less (red bars) than the predicted precipitation for a) wet initial conditions and (b) dry initial conditions. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

According to Kao and Govindaraju (2010), for computing JDI, the dimension of the selected copula model must be large enough to include the complex dependence structure of droughts. Due to the strong seasonality in the data we used maximum time window of 12 months (namely $w = 1,\ldots,12$). Therefore, we need to construct a 12-dimensional copula function to represent a 12-dimensional joint distribution from the univariate marginal distributions of each time window ($u_w^{month}, w = 1,\ldots,12$). So far, the major restriction to construct such an index has been the mathematical complication of the required 12-dimensional copula. To overcome this problem, Kao and Govindaraju (2010) adopted an empirical copula function. They showed that the analytical and empirical copulas report similar values for large datasets. Given the 32-years data record it is expected that the empirical copula will adequately construct the 12-dimensional copula for the study region.

Empirical copulas, C_n, are rank-based joint cumulative probability measures (Nelsen, 2006). A 12-dimensional empirical copula can be calculated as following:

$$C_n(u_1,\ldots,u_{12}) = \frac{1}{n}\sum_{i=1}^{n} I\left(\frac{R_{i1}}{n+1} \leqslant u_{i1},\ldots,\frac{R_{i12}}{n+1} \leqslant u_{i12}\right) \tag{4}$$

where n is the sample size; $I(A)$ denotes the indicator variable of the logical expression A and assumes a value of 0 if A is false and 1 if A is true; $R_{i1},\ldots,R_{i12}$ are the ranks of the ith observed data that are represented as $u_1,\ldots,u_{12}$, respectively, and u_w are the values of the cumulative probability of X_w^{month} where $w = 1,\ldots,12$.

A copula gives the cumulative joint probability, $P[U_1 \leqslant u_1,\ldots U_{12} \leqslant u_{12}] = s$, of the sample random variables, $\{u_1, u_2, \ldots, u_{12}\}$, which in this study represent the moisture deficit status for the different time windows. A smaller cumulative probability s would imply overall drought conditions (dry over various temporal scales), while a higher value would imply overall wet conditions. The distribution function of copulas, K_C, provides the cumulative probability function $K_C(s) = P[C_{U_1,U_2,\ldots,U_{12}}(u_1,u_2,\ldots,u_{12}) \leqslant s]$ that determines the probability of copula values that are less than or equal to s (i.e., events drier than a given threshold). K_C is the cumulative density function of $C_{U_1,U_2,\ldots,U_{12}}$, hence the definition of Joint Deficit Index (JDI) can be given as:

$$\mathrm{JDI} = \phi^{-1}(K_C(s)) \tag{5}$$

Since an analytical expression of K_C might not exist for non-Archimedean copulas, we can use an empirical distribution function K_{Cn} (Kao and Govindaraju, 2010). The empirical estimator of Kendall's distribution function, K_{Cn}, for a 12-variate sample marginal random variable is given by (Genest et al., 2009):

$$K_{Cn}(s) = \frac{1}{n}\sum_{i=1}^{n} I(\psi_i \leqslant s), \quad s \in [0,1] \tag{6}$$

where,

$$\psi_i = \frac{1}{n}\sum_{j=1}^{n} I(u_{1j} < u_{1i},\ldots,u_{12j} < u_{12i}) \tag{7}$$

and where $I(A)$ denotes the indicator variable of the logical expression A, which takes the value 0 if A is false and 1 if A is true.

Since JDI is on an inverse normal scale (same as SPI and SPI^{mod}), drought classifications similar to McKee et al. (1993) are adopted with the difference that droughts are here defined for JDI values below 0, while normal and wet conditions are for JDI values equal

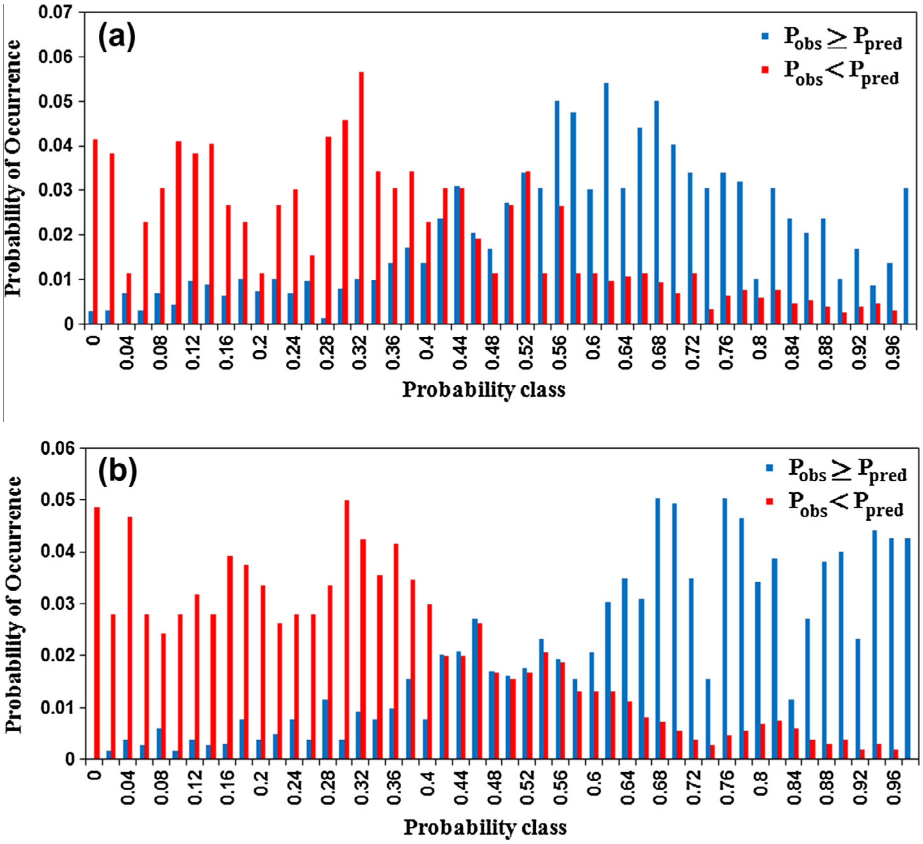

Fig. 9. Same as in Fig. 5, but for the 3-month-ahead prediction of wetness conditions.

and greater than zero, respectively. This definition follows Loukas and Vasiliades (2004) who showed that using zero as threshold for separation of dry and wet periods, gives more accurate prediction of drought conditions. Similar approach for defining drought events have been applied by other studies (e.g. Shiau, 2006; Shiau and Modarres, 2009; Mirabbasi et al., 2012).

Another advantage of the JDI index is that it allows a month-by-month future drought potential assessment. Kao and Govindaraju (2010) proposed an algorithm for estimating the required precipitation in the next months for reaching normal conditions (JDI = 0), which is described in the Appendix. The method is implemented in this study for two prediction periods (1-month and 3-month), and it is evaluated based on independent data (2002–2007) discussed in the following section.

4. Drought assessment verification

In this study, the modified SPI was calculated by fitting a two-parameter Gamma (G2) distribution on the X_w^{month} dataset as suggested by McKee et al. (1993). Model parameters were estimated using the maximum likelihood (ML) method (Zhai and Feng, 2009). In order to test the appropriateness of G2, Kolmogorov–Smirnov (KS) test was applied to determine the goodness of fit at the 5% significance level. For the 50 precipitation stations in the study region, G2 was fitted to the X_w^{month} sets of accumulated precipitation over the time windows (w) ranging from 1-month up to 12-month for calculating the modified SPI. It was found that only 41 out of 7200 sample data (50 stations × 12 window sizes × 12 months) failed to pass the KS test with the modified definitions of SPI. This indicates that G2 is an appropriate distribution for computing the modified SPI. The modified SPI and JDI were then determined for the 32-year period of rain data.

The JDI, SPI_w^{mod}, for $w = 1, \ldots, 12$ and the corresponding 12-month precipitation values are shown in Fig. 4 for four sample cases. In Fig. 4a, the SPI_w^{mod} values, determined for all time-window sizes ending in May 1987, are less than zero and depict precipitation deficits indicating a moderate to severe drought. However, in 5 out of 12 months, the observed precipitation was greater than the long-term monthly mean. The JDI reports moderate drought for May 1987. SPI_1^{mod} value is strongly negative due to the serious precipitation deficit in May. Other SPI_w^{mod} values are moderately negative, with SPI_{12}^{mod} being close to zero, due to the above-average precipitation in previous months. Despite of benefits of SPI_w^{mod} in analyzing droughts, this example shows uncertainty in using long-term SPI_w^{mod} values for identifying emerging droughts. Overall, the deficit status of this sample case was better expressed by JDI.

Fig. 4b shows an opposite case to the May 1987, in which JDI and SPI_w^{mod} depicted sufficient precipitation in all window sizes ending in December 1986. In this month, the precipitation is smaller than the long-term average, but the JDI shows a normal moisture condition. This is due to the above-average precipitation in earlier months (February–July, and October–November), which is also depicted in the SPI_2^{mod}–SPI_{12}^{mod} values. This case shows the effect of earlier precipitation that is not identifiable by the short-term SPI_w^{mod} values. However, JDI was able to express this condition well.

Fig. 4c shows a case of emerging drought in June 2002, in which SPI_1^{mod} is very negative due to the strong precipitation deficit in June, while the other SPI_w^{mod} values are greater than zero. Interpretation of this case based on SPI_w^{mod} values would be quite confusing, since most of the indices do not identify droughts in a timely manner. This case shows that the long-term SPI_w^{mod} cannot capture this emerging drought case. The JDI is less than zero which indicates dry conditions.

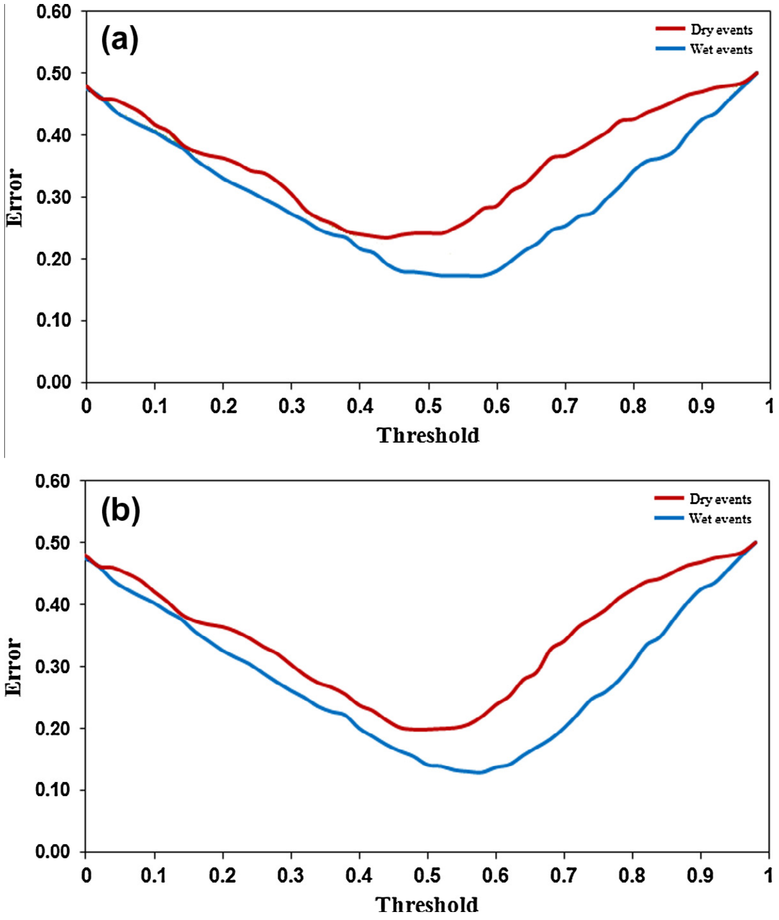

Fig. 10. Error versus exceedance probability threshold for (a) 1-month and (b) 3-month-ahead predictions.

Finally, Fig. 4d shows a prolonged drought in August 2005, in which SPI_1^{mod} and SPI_2^{mod} indicate sufficient precipitation in August and July, while other SPI_w^{mod} values are less than zero due to precipitation deficit in prior months. The JDI reports a moderate drought, which reflects the effect of moisture deficits in earlier months. The above cases indicate that the JDI can capture drought with less uncertainty than SPI_w^{mod}. The contingency table approach is devised here to quantify the performance of JDI and SPI_w^{mod} indices in identifying emerging and prolonged drought events (Gandin and Murphy, 1992). Specifically, the contingency table is used to show the frequency of "yes" and "no" predicts and actual occurrences. The four combinations of predictions (yes or no) and observations (yes or no) are: Hit: event forecast to occur, and did occur, Miss: event forecast not to occur, but did occur, False alarm: event forecast to occur, but did not occur, and Correct negative (correct nonevents): event forecast not to occur, and did not occur. A variety of categorical statistics are used to describe aspects of forecast performance based upon the contingency table (Wilks, 1995). The skill score is expressed as the ratio of the hits and misses with respect to the possible totals. Among these, the Critical Success Index (CSI) (Donaldson et al., 1975), Probability of Detection (POD) and False Alarm Ratio (FAR) are commonly used in forecast verification. The CSI, POD and FAR are defined as

$$\mathrm{CSI} = \frac{hits}{hits + misses + false\ alarms} \tag{8}$$

$$\mathrm{POD} = \frac{hits}{hits + misses} \tag{9}$$

$$\mathrm{FAR} = \frac{false\ alarms}{hits + false\ alarms} \tag{10}$$

The contingency tables of SPI_1^{mod}, SPI_6^{mod}, SPI_{12}^{mod} and JDI for emerging and prolonged drought events from the 30-year data record are given in Tables 1 and 2, respectively. An emerging drought event is counted if observed precipitation was greater than the long-term monthly mean in previous month and it is less than the long-term monthly mean in current month. On the other hand, if observed precipitation in 12 consecutive months was less than the long-term monthly mean, then it is counted as a prolonged drought event.

The corresponding values of CSI, POD and FAR are given in Table 3. It can be seen that the JDI and SPI_w^{mod} indices exhibit better performance in identifying prolonged droughts relative to emerging events. In particular, the long term SPI_w^{mod} cannot identify an emerging drought. In the case of prolonged droughts the JDI shows better performance than all SPI_w^{mod} indices. Fig. 5 also shows CSI values of SPI_w^{mod} ($w = 1, \ldots, 12$) and JDI for the emerging and prolonged drought events. It is shown that for emerging droughts the highest value of CSI belongs to SPI_1^{mod} and the CSI value for JDI is slightly lower than SPI_1^{mod}. In the case of prolonged drought the highest CSI value is with JDI, while SPI_3^{mod} exhibit close performance. These results confirm that single JDI index can provide more efficient detection of both emerging and prolonged drought events, as reported by Kao and Govindaraju (2010).

Illustration of JDI time series using marginal precipitation variables from the Urmia station for the period 1970–2007 is shown in Fig. 6. This station is located besides Urmia Lake. As noted by the

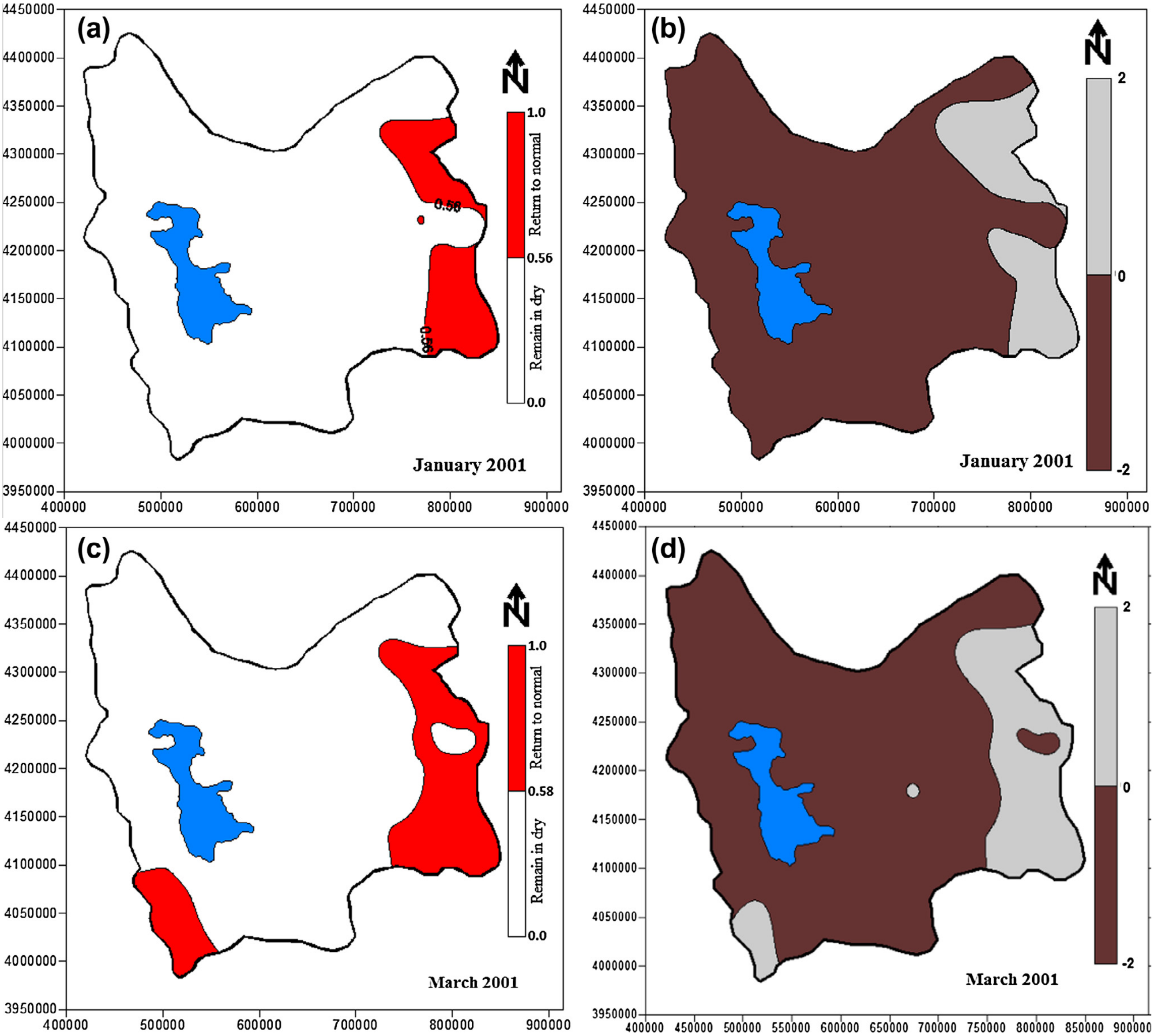

Fig. 11. The predicted exceedance probability of required precipitation to achieve normal status (JDI = 0) in (a) January 2001 and (c) March 2001, and the corresponding JDI based on observed precipitation up to (b) January 2001 and (d) March 2001.

JDI time series after 1997 this area experienced a prolonged (~10 years) and severe (JDI < −1) drought. Indeed due to the drought and the associated increased demands for agricultural water of the lake's basin, the Urmia lake level decreased during that period by about 4.5 m as reported by the West Azarbaijan Environment Organization. Also, the salinity of the lake rose to more than 300 g/L and large areas of the lake bed were desiccated (Eimanifar and Mohebbi, 2007).

5. Prediction of wetness conditions

As mentioned above the method can also be used for a month-by-month future drought potential assessment. Fig. 7a shows a regional illustration of the precipitation for January 2001 required to achieve normal status (JDI = 0), calculated using the algorithm proposed by Kao and Govindaraju (2010). Fig. 7b shows the observed precipitation for January 2001. In January 2001, with the exception of small parts in the east and west of the region the majority of the area received very low precipitation. The regional illustration of JDI prior to January 2001 is shown in Fig. 7c. The period of 1997 to 2002 was one of the worst droughts in the north-west of Iran, which is confirmed by the JDI values shown in Fig. 7c. According to Fig. 7a, to return to normal condition the majority of the study area had to receive 50 mm of rain in January 2001, while historical January precipitation data show a climatological monthly average of about 30 mm in most of the considered stations. The exceedance probability of the above precipitation value required to reach normal status (JDI = 0) is shown in Fig. 7d. As shown by this probability map the chances of drought recovery in January 2001 were determined to be small in most parts of the region (less than 0.3). For example at Urmia station, it required 46 mm of precipitation to bring JDI back to normal, which is about 1.5 times greater than the climatological average for that month (29.8 mm). The exceedance probability shows that the chance of drought recovery in the following month was less than 0.2. The actual precipitation in January 2001 was 25.6 mm, which confirmed the persistence of drought.

Fig. 8 shows frequency plots of the exceedance probabilities when we start from drought or normal/wet conditions, and the

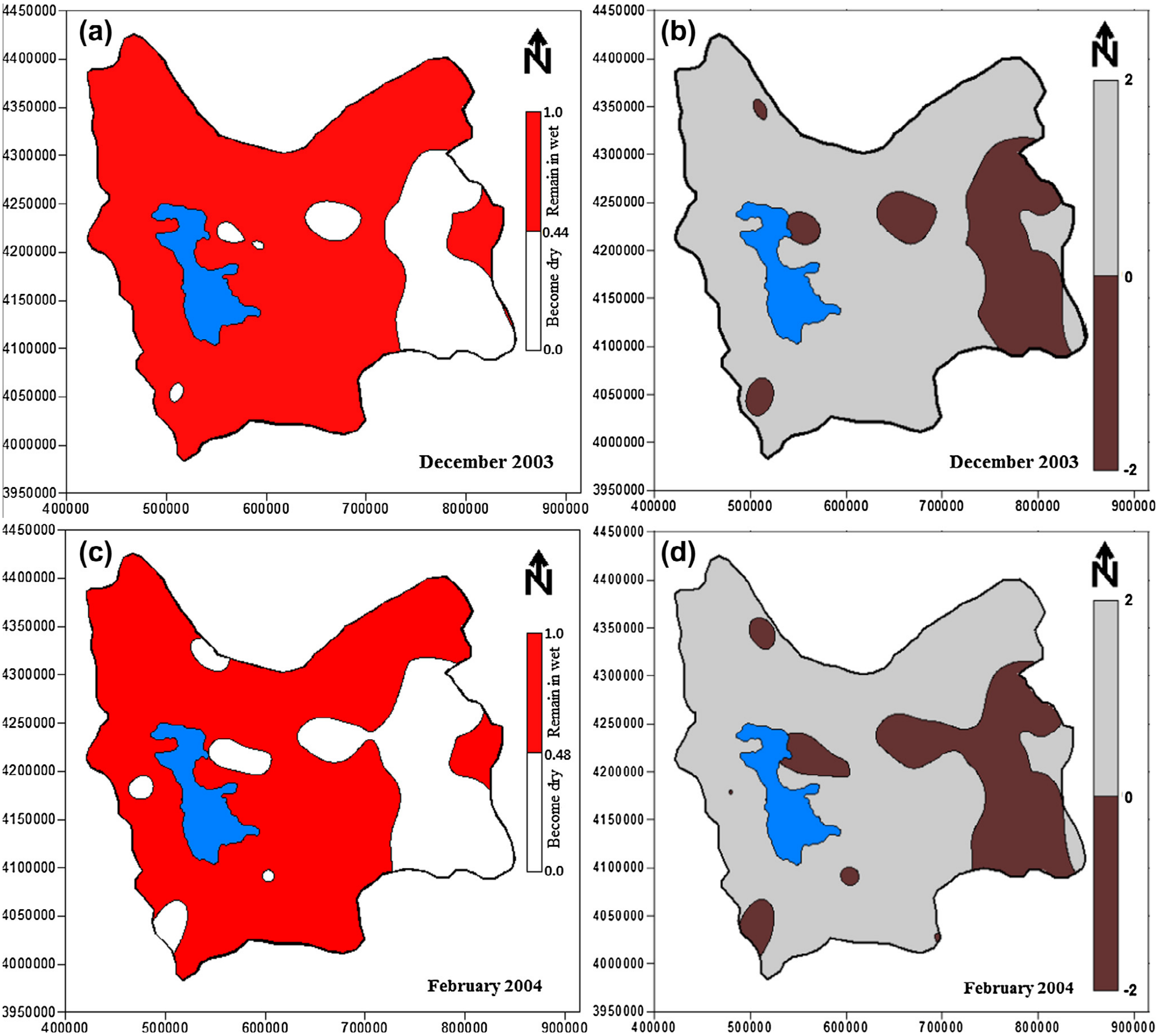

Fig. 12. The predicted exceedance probability of required precipitation to achieve normal status (JDI = 0) in (a) December 2003 and (c) February 2004, and the corresponding JDI based on observed precipitation up to (b) December 2003 and (d) February 2004.

subsequent month is at normal/wet conditions (blue bars) or dry condition (red bars). Fig. 9 shows the same frequency plots but relating to conditions in 3-month ahead. The figures are based on data from 2002 to 2007. Because of the persistence in moisture conditions we considered wet and dry months separately. Namely, when the JDI for present month was positive (wet conditions) we calculated the minimum precipitation value required for remaining at normal moisture condition in the subsequent month (or next 3 months) and the associated probability of exceedance based on the Kao and Govindaraju (2010) method described in the Appendix. In a similar manner, when the JDI for the present month was negative (dry conditions) we calculated the minimum precipitation required for reaching normal/wet moisture condition in the subsequent month (or next 3 months) and the associated probability of exceedance. The figures show that higher exceedance probability classes are associated with higher occurrence of normal/wet conditions in the next month (or 3 months), and vice versa low exceedance probabilities are related to more frequent dry conditions in the next month (or 3 months). The figures also show that the two frequency plots have some degree of overlap, which indicates that for the same exceedance probability we can have occurrences of both conditions (normal/wet and dry) in the subsequent month (or 3 months).

To determine an optimum exceedance probability threshold for predicting the drought conditions of future months based on JDI we devised an error function that quantifies the frequency of false detections and detection failures of the actual wetness condition. False detection occurs when the predicted exceedance probability is greater than the specified threshold, but the observed precipitation is lower than the predicted precipitation ($P_{\text{obs}} < P_{\text{pred}}$). Detection failure occurs when the predicted exceedance probability is below the specified threshold, but the observed precipitation is greater or equal than the predicted precipitation ($P_{\text{obs}} \geqslant P_{\text{pred}}$). The error function was evaluated for different exceedance probability threshold values based on data from the verification period (2002–2007). Fig. 10 shows the variation of error versus the exceedance probability threshold for the 1-month (left panel) and 3-month (right panel) wetness condition predictions for the two initial states (dry and wet). For the 1-month-ahead prediction, the optimum exceedance probability thresholds for wet and dry

Table 4
Contingency table of predictions with dry initial conditions (thresholds = 0.44/0.48 for 1-month/3-month).

Predicted	Observed		
	Yes (1-mn/3-mn)	No (1-mn/3-mn)	Total (1-mn/3-mn)
Yes	543/624	188/169	731/793
No	142/131	1234/1221	1376/1352
Total	685/755	1422/1390	2107/2145

Table 5
Contingency table of predictions with wet initial conditions (threshold = 0.56/0.58 for 1-month/3-month).

Predicted	Observed		
	Yes (1-month/3-month)	No (1-mn/3-mn)	Total (1-mn/3-mn)
Yes	651/688	202/197	853/885
No	144/151	496/419	640/570
Total	795/839	698/616	1493/1455

Table 6
CSI, POD and FAR determined from the contingency tables of Tables 4 and 5.

Skill score	Wet events (1-mn/3-mn)	Dry events (1-mn/3-mn)	Total events (1-mn/3-mn)
CSI	0.653/0.664	0.622/0.675	0.609/0.635
POD	0.818/0.820	0.793/0.826	0.769/0.770
FAR	0.237/0.223	0.257/0.213	0.254/0.216

conditions are 0.44 and 0.56, respectively. For the 3-month-ahead predictions the corresponding optimum thresholds are slightly higher (0.48 and 0.58, respectively). Comparing values of error and optimum probability thresholds for the 1-month and 3-month predictions indicates that for the longer prediction period, the error slightly decreases for both categories. Furthermore, we note that dry initial condition exhibit larger error than the wet initial condition.

Fig. 11a shows a map depicting the exceedance probability of the required precipitation to achieve normal status for a sample case in January 2001, determined based on the JDI of December 2000 (i.e. 1-month prediction). December 2000 was a dry month, thus Fig. 11a is based on the exceedance probability threshold of 0.56. According to the figure, it is expected that large part of the study area would persist with dry condition in January 2001, which is confirmed by comparing with the JDI determined based on precipitation observations up to January 2001 (Fig. 11b). Similarly, Fig. 11c shows the exceedance probability of the required precipitation to achieve normal status in the next 3-months (March 2001). For the 3-month case and dry initial conditions the optimal threshold has been determined to be 0.58. The figure shows that large part of the study area would experience dry condition in March 2001, which is also confirmed by the agreement with the JDI determined based on precipitation observations up to March 2001.

Fig. 12 shows a wet initial condition sample case. The November 2003 was a wet month, so we used 0.44 and 0.48 as the exceedance probability threshold values for predicting the 1-month-ahead (December 2003) and 3-month-ahead (February 2004) drought conditions, respectively. The figure shows that large part of the study area will remain in wet condition through December 2003 (Fig. 12a) and February 2004 (Fig. 12c). As in the previous sample case, JDI determined based on observed precipitation data shows close agreement with the predicted wetness conditions.

The contingency table approach is applied here to quantify the performance of the 1-month-ahead and 3-month-ahead wetness condition predictions. Specifically, the contingency tables for dry and wet initial conditions are given in Tables 4 and 5, respectively. The corresponding performance metrics of CSI, POD and FAR are given in Table 6. The CSI values ranged from 0.609 to 0.675, which shows a good prediction skill for the method. The POD ranged from 0.769 to 0.826 indicating that more than 75% of the observed events were correctly predicted by the method. The FAR is between 0.213 and 0.257, indicating that less than 25% of the predicted prolonged droughts did not occur.

6. Conclusions

An assessment of meteorological droughts in the northwest of Iran was conducted based on monthly precipitation data from 50 rain gauge stations in the period 1970 to 2007. First, time-series of the modified Standardized Precipitation Index (SPI^{mod}) were constructed for different periods based on monthly mean precipitation values, to determine drought categories. Then a regional analysis of meteorological drought was provided by means of JDI for northwest of Iran. Because of the existence of strong seasonality in the precipitation data of all selected stations in the region, using the SPI method would yield confusing results. Therefore, application of the modified SPI and JDI at all of gauge stations yielded more accurate characterization of drought conditions than SPI.

The data analysis showed that the JDI could identify well both emerging and prolonged droughts. On the other hand, the short-term (long-term) SPI_w^{mod} values could not identify prolonged (emerging) droughts. JDI expressed the overall deficit status in terms of the joint cumulative probability; hence, it provided a more comprehensive assessment of the drought condition. It was also shown that JDI can allow a month-by-month drought assessment such that the required amount of precipitation for achieving normal conditions in the subsequent months can be determined. The performance of this approach was evaluated based on 6 years (2002–2007) of independent rain gauge rainfall data. The results of the error analysis showed that the method can provide accurate 1-month-ahead and 3-month-ahead wetness condition predictions. Specifically, the critical success index of the drought conditions for the 1-month and 3-month predictions were 0.609 and 0.635, respectively. This indicates a good performance and slight improvement in the prediction accuracy for longer prediction periods (3 months). The spatial maps also showed close agreement between the predicted wet/dry conditions and the JDI determined from observed precipitation.

The method can provide valuable information for water resources planners and policy makers in developing appropriate management to cope with drought consequences. Future extension of this work would be to use the JDI for better estimation of drought return periods, determining the persistence of droughts at a given severity for risk assessment. Furthermore, this study was limited to predicting the drought conditions up to 3-months ahead. As mentioned in Appendix, we can extend the predicted drought conditions up to 12 months, which need to be evaluated based on future studies.

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Appendix A. Calculating the required precipitation to return to normal conditions

In this appendix we describe the Kao and Govindaraju (2010) procedure proposed to calculate the required precipitation to return to normal conditions in the next month. Let P_w^o denote the observed monthly precipitation for the past w months, and P_n^f represent the future n-month monthly precipitation to be assessed. Since the maximum time window length used in JDI is 12 months, in order to assess P_1^f one needs to have the past 11-month observations $(P_1^o, P_2^o, \ldots, P_{11}^o)$. Since P_1^f is unknown, the precipitation marginal distributions (u_i^{month}) cannot be determined directly (e.g., u_1^{month} is controlled by P_1^f, and u_2^{month} is controlled by $P_1^f + P_1^o$). Therefore, to solve for the value of P_1^f associated with normal conditions (JDI = 0), one may follow the procedure listed below:

1. Assign an initial guess of P_1^f.
2. Compute the marginal distributions u_1^{month} by P_1^f, u_2^{month} by $(P_1^f + P_1^o), \ldots, u_{12}^{month}$ by $\left(P_1^f + \sum_{i=1}^{11} P_i^o\right)$.
3. Compute the 12-dimensional empirical copula, $C_{U_1,\ldots U_{12}}(u_1^{month}, \ldots, u_{12}^{month})$, and the corresponding K_C values.
4. If K_C is different than 0.5 then the process repeats by adjusting P_1^f otherwise the procedure has converged to the precipitation value (P_1^f) that associates with normal conditions ($K_C = 0$ or JDI = 0). Namely, P_1^f represents the required precipitation over the following 1 month in order for the joint deficit status to be normal, and $(1 - u_1^{month})$ will be the exceedance probability of this event. The required precipitation along with its exceedance probability provides good interpretation of the drought status in future month(s).

Kao and Govindaraju (2010) also generalized the above procedure for estimating the required total precipitation in future n-months $(2 \leqslant n \leqslant 12)$ for achieving normal conditions. In these cases, the corresponding exceedance probability is equal to $(1 - u_n^{month})$.

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