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Using the pendulum column as an isolator by reducing the gravity effect

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Abstract. The conventional method of structural seismic design was based on increasing structural capacity, which usually didn't reduce earthquake seismic effects. By changing the philosophy of structure design, technologies such as passive seismic control have been used in structures. So far, a large number of seismic isolation systems have been introduced to dissipate earthquake energy that is applied to a structure. These systems act against earthquakes rather than increasing the strength and capacity of the structure. In the present paper, a suspended column called a "pendulum column" is investigated, and a new idea has been considered to improve the performance of the pendulum column isolator by changing the gravity effect by adding a spring under the isolator system. The behavior of the studied isolator system has been researched. Then the isolator system was investigated under different earthquakes and compared with a common pendulum column isolator. The results show that changing the gravity effect has an effective role in the response of the system by reducing the system stiffness. Equations for the system showed that even in a special state, complete isolation is possible. Finally, the tested model verified the theory.

Keywords: damping; earthquake; effective gravity; isolator; pendulum column

1. Introduction

The conventional methods of structural seismic design are based on increasing structural capacity. In this seismic design approach, the provision of lateral loading capacity in the structure is done by increasing its strength and providing its ductility. As a result of the execution based on this approach, the structural dimensions and connections increased. It is difficult to control the occurrence of earthquake damage. In the event of severe earthquakes, constructed structures experience significant acceleration. This will ultimately lead to a loss of comfort for residents of buildings and damage to structural and non-structural components. By changing the philosophy of structure design, technologies such as passive seismic control of structures have been used. By using isolators, the dynamic behavior of the structure can be predicted, and the amount of seismic damage to structural and non-structural components is reduced. The main objective of a seismic isolation method is to prevent the direct transfer of earthquake force from the foundation to the structure. As a result of the seismic isolation, the natural period of the structure increases. Due to the increase in the natural period, the acceleration exerted by the earth's motion on the structure decreases.

From the review of the current approach to date, the general principles are for buildings or structures to be decoupled from the horizontal components of the earthquake ground motion by interposing a layer with low horizontal stiffness between the structure and the foundation. The harmonious movement of the structural basement will cause a significant reduction in the fundamental frequency, which is much lower than its fixed-base frequency and also much lower than the predominant frequencies of the ground motion (Monfared *et al.* 2013).

In Iran, where it is one of the most seismically active regions of the world, some historical buildings in Pasargadae, which date back to at least 2500 years ago, have lasted without seismic damage to date. In those monuments, multi-layer stones have been used as a construction method. The monuments contain large stones with smooth and flat surfaces. They have less friction during an earthquake's excitation and are able to move back and forth over the lower foundation without damage (Monfared *et al.* 2013). Kelly (1986) provided extensive reviews of historical developments and literature on the base isolation. Similar examples, which date back to the 15th century, can be found in the dry-stone walls of the Machu Picchu Temple of the Sun in Peru (Wright 2000). A base-isolated structure was proposed by Kawai in 1988. In 1909, a seismic isolation system was proposed by Johannes Calantarients (Naeim and Kelly 1999). Johannes Calantarients suggested separating the structure from the foundation with a layer of talc (Islam 2011). Rubber bearings as a base isolation were used in a primary school in Yugoslavia in 1969 (Izumi 1988). A sliding (SR) resilient base isolation system was discussed by Su *et al.* (1991). The isolator peak response was not too serious in frequency and amplitude content. The seismic results of rigid bases, base-isolated concentrically braces, and special moment-resistant steel frames were studied by Lin *et al.* (1992). Shenton *et al.* (1993) performed nonlinear dynamic analysis for fixed-base and base-isolated structures by choosing three different types of time histories: post-earthquake records. Barghian and Shahabi (2007) introduced pendulum base isolation.

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The stiffness of their base isolation was settable; therefore, the stiffness could be set in a way that, first, the force applied to the structure would be acceptable, and second, the maximum displacement would not exceed the allowable limit. Concellara *et al.* (2013) described the difference between the lead rubber isolator and friction slider. Then an isolator composed of a lead rubber bearing and friction slider (named the high-damping hybrid seismic isolator, HDSI) was studied. The seismic response of the HDSI was different from the lead rubber isolator response. A structure was examined under different levels of seismic activity in terms of frequency and intensity. Results were compared in the form of base shear, shear force, and displacement at the base of the structure. The comparative result showed that the HDSI provided superior safety for severe seismic activity compared to either the lead rubber isolator or friction slider. Luco *et al.* (2014) discussed the interaction effect of soil structure on a base-isolated building. The results showed that the deformation of an inelastic structure was high when the effect of soil was taken into account. When the interaction of the structure of the soil was ignored and an undamped vibration was considered, critical harmonic excitation occurred. The results obtained depended on the damping of the isolator. Li *et al.* (2014) studied a new base isolator with variable stiffness and damping. As the impact of earthquakes is sometimes so serious that the passive nature of rubber is not able to generate energy due to seismicity, smart base isolation with adaptive and controllable properties was developed by them, with different stiffness and damping properties of the isolator. Shrimali *et al.* (2015) discussed that the use of control devices for seismic vulnerability was increasing rapidly. Their study focused on hazardous reasons due to the pounding effect of nearby buildings. They concluded that, in order to decrease this damage, the use of controlled devices had become essential. Their study was based on a comparative analysis of damper and isolated systems. Thomas and Mathai (2016) studied statically the Friction Pendulum System (FPS). The FPS acted like a bearing. The superstructure was isolated from the foundation using specially designed concave surfaces and bearings to allow sway under its own natural period during the seismic events. Xiuting *et al.* (2018) proposed an isolation and protection structure inspired by the structure and forcing method of leg parts of bipeds, the Origami-Joint Flexible (OJF) isolation structure. In the following, a new method called effective gravity applied to the pendulum isolator to control the seismic effect is proposed. The pendulum isolator is a base isolator that is mounted on the foundation and is not a TMD system. Shah and Soni (2017) conducted a study on a single-story building that was isolated using a Double Concave Friction Pendulum (DCFP) system. They varied the friction coefficients and examined the behavior of the top and bottom sliding surfaces when subjected to triaxial ground motions compared to unilateral and bilateral ground motions. The results showed that the triaxial ground motion had a significant impact on the building's response compared to the unilateral ground motion. Cirelli *et al.* (2019) proposed a new modeling approach and design procedures for the trapezoidal bifilar centrifugal pendulum vibration absorber, which is used to mitigate torsional vibrations in rotating machinery. Lupăşteanu *et al.* (2019) implemented a base isolation method to rehabilitate an ancient building. They used friction pendulum sliding (FPS) isolators to decouple the superstructure from the existing foundation system and transfer it to the seismic isolators. Deringöl and Güneyisi (2019) investigated the influence of friction pendulum bearing (FPB) isolator characteristics on the nonlinear response of buildings under different seismic excitations. They found that adjusting the isolation period, yield strength ratio, and effective damping ratio accurately estimated the seismic response of base-isolated frames. Zhong *et al.* (2022) proposed a self-centering base mechanism for high-rise buildings to mitigate higher-mode effects and control residual deformations and concentrated stresses within the structure. Their numerical case studies confirmed improved seismic performance with minimal damage and residual deformations after major seismic events. Kim *et al.* (2022) analyzed the performance of a 3D office building frame with base isolation and a bracing system using nonlinear time history analysis. The presence of rubber friction bearings (RFB) significantly reduced story drift and frame damage during earthquakes. Auad *et al.* (2022) assessed the seismic reliability of nonlinear base-isolated structures equipped with Lateral Impact Resilient Double Concave Friction Pendulum (LIR-DCFP) components. They developed fragility curves based on probabilities exceeding specific limit states and concluded that structures with LIR-DCFP devices exhibited better seismic performance compared to classical DCFP bearings. Castaldo *et al.* (2022) studied isolated multi-span continuous deck bridges, considering the influence of friction pendulum isolators. They incorporated uncertainties in the seismic input and friction coefficient to evaluate fragility curves for both the pier and the isolation system supporting the deck. They proposed seismic reliability-based design algorithms for different structural properties. Gino *et al.* (2023) investigated the seismic reliability of multi-span continuous deck bridges with isolation friction pendulum (FP) devices. They considered the uncertainties associated with the sliding friction coefficient of the FP isolators and used scaled natural seismic records to determine fragility curves for the RC pier and isolator devices. Seismic reliability curves in the performance domain were obtained using the convolution integral.

Paolo Castaldo and Elena Miceli (2023) investigated the analysis of the optimal friction coefficient with respect to the seismic performance of isolated multi-span continuous deck bridges -a six-degree-of-freedom system- equipped with single concave friction pendulum (FPS) isolators. They considered the peak ground acceleration-to-velocity ratio (PGA/PGV) and the peak ground acceleration (PGA) as ground motion parameters and showed the effectiveness of the PGA/PGV ratio within the proposed nondimensionalization, together with the existence of an optimum value of the friction coefficient that minimizes the nondimensional response of the pier. Li *et al.* (2021) investigated the use of a base isolation system-tuned mass damper inerter (BIS-TMDI) system to mitigate the displacement demand of the isolation layer during strong

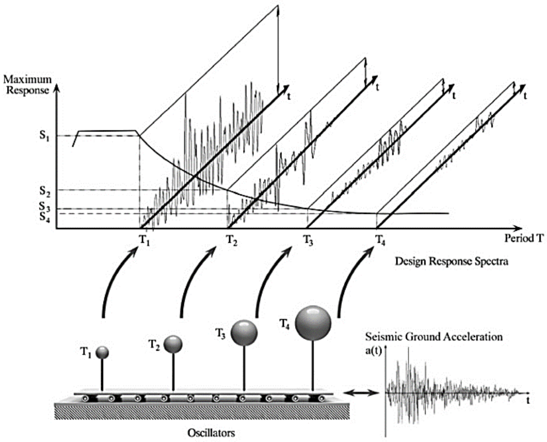


Fig. 1 Responses spectrum (Vrochidou *et al.* 2014)

earthquakes. The researchers found that the nonlinear BIS-TMDI system was able to effectively reduce the displacement demand of the isolation layer. Furthermore, they observed that its control effectiveness and stroke performance are better than those of the nonlinear BIS-TMD system. Chen *et al.* (2023) investigated the seismic-resilient design of tall bridge piers with rocking foundations using inerter-based systems. They developed a simplified model in OpenSees to simulate a rocking bridge pier that was equipped with a tuned viscous mass damper (TVMD) system. They utilized an equivalent model that consisted of a rotational mass linked with the two terminals of the TVMD through appropriate constraint conditions for modeling the inerter-based system. The parametric analysis results indicated that the TVMD was found to be less efficient for near-fault motions. This is due to a reduced energy dissipation capacity caused by the pulse-type feature. Cao and Li (2023) proposed a novel hybrid passive base-isolated system integrating the base-isolated system (BIS) with the tuned tandem mass damper inerters (TTMDI). modeled a simplified four-degree-of-freedom to reveal the interaction between two components and their integrated control performance. Results confirmed that the BIS+TTMDI has a high control effectiveness, high robustness, a highly smaller stroke, and drastically reduced damping demand.

2. The governing equations of the studied model

Seismic isolation is an engineering method used to reduce the effects of earthquakes on structures. The main purpose of this method is to isolate the structure from ground motion and transfer seismic forces to it. One of the important aspects of seismic isolation is increasing the periodicity of the structure.

By increasing the periodicity, the structure responds more gently to ground movement and absorbs seismic energy more effectively. Seismic forces are transferred to the structure in a non-impact way; also, by adding a damper, the possibility of resonant vibrations that can lead to damage and failure of the structure is reduced, which

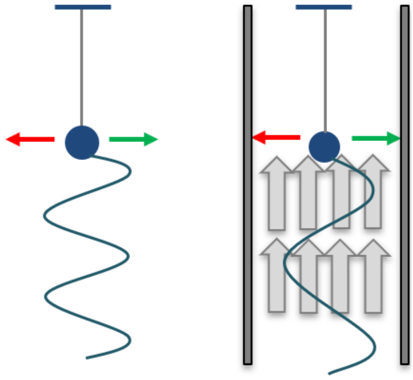


Fig. 2 The effect of vertical load on pendulum responses (Azizi and Barghian 2015)

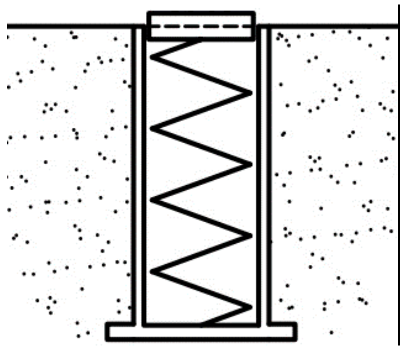


Fig. 3 Compressed spring system

reduces the stresses and unstable deformations in the structure.

The effect of increasing the periodicity on the response of the structure can be seen in Fig. 1.

However, the ability of seismic isolators to increase periodicity is limited. For example, due to friction on the surface of fps isolation, increasing the radial area of the surface causes the behavior of the isolator to be non-periodic. Or, in the pendulum column (Azizi and Barghian 2023a), extra increases in the length of the pendulum are impossible. To improve the behavior of this isolation, a supplement can be defined. The concept of this supplement refers to simple pendulum behavior in the air tunnel, as shown in Fig. 2. In the vertical tunnel and with the air flow, the period of the pendulum is affected. In this case, the air flow passes from the bottom to the top in the tunnel, and as a result, a force is applied to the pendulum. By reducing the effect of gravity, the periodicity of the pendulum increases.

To implement this concept, by designing a cylinder with a compacted spring in it, a vertical force can be obtained. To have planar freedom, two slider plates can be defined. (Fig. 3).

For isolators like fps (Fig. 4) or pendulum columns (Figs. 5-6) that have pendulum behavior, this system can be combined.

Fig. 7 shows a 3D view of the pendulum column. As it is seen from Fig. 7, at the pendulum column, the ∨ and ∧ elements are perpendicular to each other and are connected to each other by a two-ended hinged rod. The ceiling is hung by the ∨ elements.

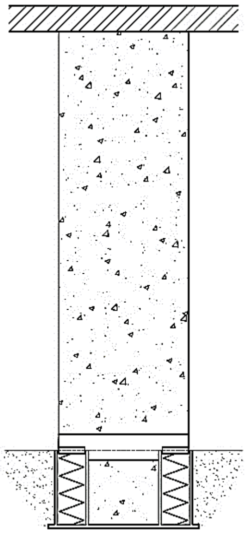


Fig. 4 The combination of an Fps isolator with a compressed spring

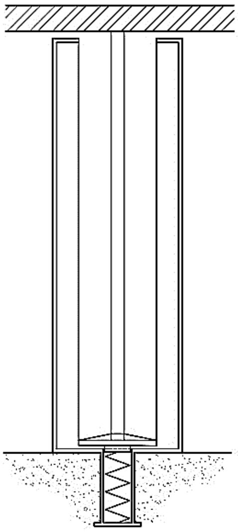


Fig. 5 The combination of a cylindrical pendulum column isolator with a compressed spring

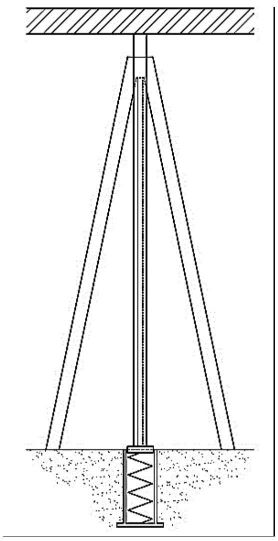


Fig. 6 The combination of a pendulum column isolator with a compressed spring

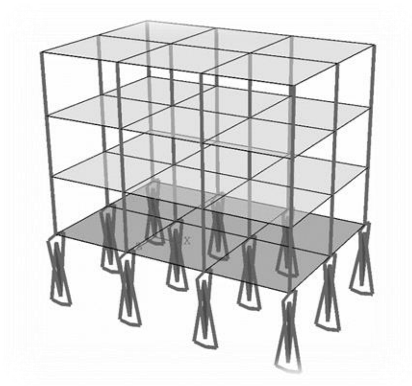


Fig. 7 Schematic 3D picture of the studied method

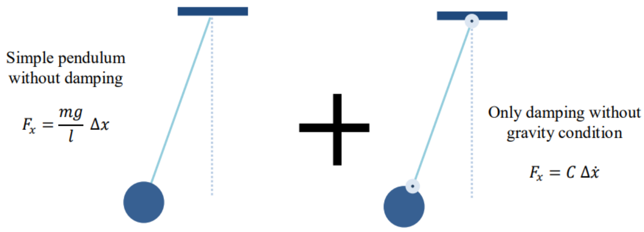


Fig. 8 The acceleration of the weight when the load is applied to the mass

To investigate the effect of vertical force, a simple pendulum is assumed; a one-hung weight pendulum is considered to explain the pendulum isolator. When the weight of the pendulum deviates from its original location, it returns to its original point after several oscillations. If the upper part of the pendulum (the support) moves instead of the lower part in the horizontal direction, then a force and an acceleration are applied to the weight.

An increased vertical spring reduces the re-centering force, and consequently, the time period of the system increases. The point is that, for earthquake-like movements, the acceleration of the weight is much less than the acceleration of the upper part of the pendulum. The image of this concept is shown in Fig. 8.

Since the isolator is equivalent to a pendulum, the equations are written for the pendulum in two simple and modified forms.

In the following equations, the letters "*e*" and "*s*" refer to the earth and the structure's mass, respectively. At first, the earth is assumed to be displaced by x_e, and the lower part of the pendulum is displaced by x_s horizontally. In this case, an angle "θ" is created in the rod, which causes the force to be applied to the weight.

The created angle 'θ' is defined as follows

$$\theta = \frac{x_s - x_e}{l} = \theta_s - \theta_e \tag{1}$$

From mechanics, the kinetic energy and potential energy are written as Eqs. (2) and (3), respectively, where, m, l and v_s are the mass, length, and velocity of the pendulum, respectively. "h" is the height of the pendulum from its equilibrium state.

$$E_{kin} = \frac{1}{2}mv_s^2 = \frac{1}{2}ml^2\dot{\theta}_s^2 \tag{2}$$

$$E_{pot} = mgh = mgl(1-\cos(\theta)) = \frac{1}{2}mgl\theta^2$$
$$cos\theta = \sum_{n=0}^{\infty}\frac{(-1^n)\theta^{2n}}{(2n)!} = 1-\frac{\theta^2}{2!}+\cdots \tag{3}$$

As the system acts like a spring, the external work is defined as follows

$$w_{ext} = \int_0^t p\dot{x}_e\, dt,\ p = \frac{mg}{l}(x_s - x_e) \tag{4}$$

By differentiating the equations, it can be written

$$dE_{pot} = mgl\theta\dot{\theta}dt$$
$$dE_{kin} = ml^2\dot{\theta}_s\ddot{\theta}_s dt \tag{5}$$
$$dw_{ext} = p\dot{x}_e dt$$

$$dE_{kin} + dE_{pot} + dw_{ext} = 0 \tag{6}$$

$$ml^2\dot{\theta}_s\ddot{\theta}_s dt + mgl\theta\dot{\theta}dt + p\dot{x}_e dt = 0 \tag{7}$$

Inserting Eq. (1) into Eq. (7) results in

$$m\dot{x}_s\ddot{x}_s + \frac{mg}{l}(x_s - x_e)(\dot{x}_s - \dot{x}_e) + \frac{mg}{l}(x_s - x_e)\dot{x}_e = 0 \tag{8}$$

Finally,

$$\ddot{x}_s + \frac{g}{l}(x_s - x_e) = 0 \tag{9}$$

In order to avoid the resonance of the structure during the earthquake, the length of the pendulum can be increased due to the relationship between the period and gravity as follows (Eq. (10))

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{10}$$

However, the length "*l*" has a limit in real structures. Therefore, in this research, it has been tried to change the "*g*" to the "effective *g*", for the first time to change the natural period. To change the gravity effect, a spring has been assumed from the bottom of the pendulum (Fig. 9). By inserting a spring under the isolator, the period of the system is increased. In this case, the external work will be defined as

$$p = \frac{mg-k\left(\Delta - l\frac{\theta^2}{2}\right)}{l}(x_s - x_e)$$
$$w_{ext} = \int_0^t p\dot{x}_e\, dt = \int_0^t \frac{mg-k\left(\Delta - l\frac{\theta^2}{2}\right)}{l}(x_s - x_e)\dot{x}_e\, dt \tag{11}$$

Where

$$dw_{ext} = p\dot{x}_e dt = \frac{mg-k(\Delta - l\frac{\theta^2}{2})}{l}(x_s - x_e)\dot{x}_e dt \tag{12}$$

In this equation, "*Δ*" is the initial compressed length of the spring due to creating an uplifting force under the pendulum, and "*k*" is the stiffness of the spring.

The energy of the spring, which is located under the isolator, will be defined as

$$E_{spr} = \frac{1}{2}k\left(\Delta - l\frac{\theta^2}{2}\right)^2 \tag{13}$$

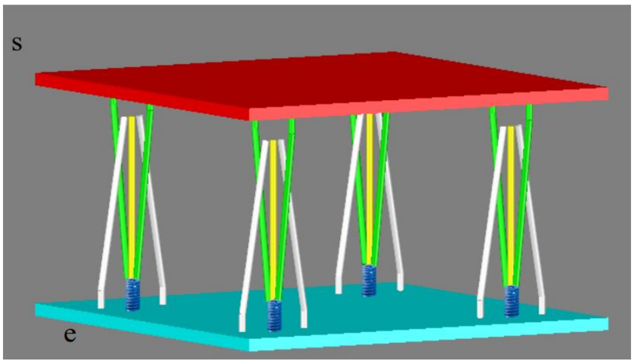


Fig. 9 Schematic 3D picture of springs connected to the floor

Differentiating Eq. (13) yields

$$dE_{spr} = -k\left(\Delta - l\frac{\theta^2}{2}\right)l\theta\dot{\theta}dt \tag{14}$$

By considering Eq. (6) again and adding Eq. (14) to it, we get Eq. (15).

$$dE_{kin} + dE_{pot} + dE_{spr} + dw_{ext} = 0 \tag{15}$$

$$\ddot{x}_s + \frac{g}{l}\left(1 - \frac{k\left(\Delta - l\frac{\theta^2}{2}\right)}{mg}\right)(x_s - x_e) = 0 \tag{16}$$

The term $\left(1 - \frac{k\left(\Delta - l\frac{\theta^2}{2}\right)}{mg}\right)$ is due to the application of a spring at the bottom of the isolation system.

In order to apply damping force, it is assumed that there is no spring or gravity in the system. According to the simplification in Fig. 8, since the main deformation is in the joints of the middle rod of the proposed isolator, assuming rigid piers and fixed supports, the damping will be only in the nodes *e*, and *s*. In this case, if the support starts to move, the weight will accelerate. By computing momentum around the middle bar nodes at the top and bottom of the tensile rod, Eqs. (17) and (18) can be written as follows

$$m\ddot{x}_s l - C_s\left(\frac{\dot{x}_e - \dot{x}_s}{l}\right) = C_e\left(\frac{\dot{x}_e - \dot{x}_s}{l}\right) \tag{17}$$

$$\ddot{x}_s = \left(\frac{C_s + C_e}{m.l^2}\right)(\dot{x}_e - \dot{x}_s) = \frac{C}{m}(\dot{x}_e - \dot{x}_s) \tag{18}$$

Finally, the Eq. (19) can be written by combining Eqs. (16) and (18).

$$\ddot{x}_s + \frac{C}{m}\dot{x}_s + \frac{g_{eff}}{l}x_s = \frac{C}{m}\dot{x}_e + \frac{g_{eff}}{l}x_e \tag{19}$$

Where, "g_{eff}", is the effective gravity.

An effective gravity method can be coupled with many isolation systems.

It is noted that geometric non-linearity was assumed by considering the p-delta effect. This base isolation system exhibits a pendulum-like behavior that is influenced by gravity, and its performance during an earthquake is dependent on changes in the internal rod angle and the gravity reducer spring located at the bottom of the system. These changes result in non-linear behavior during seismic events, as shown in Eq. (16).

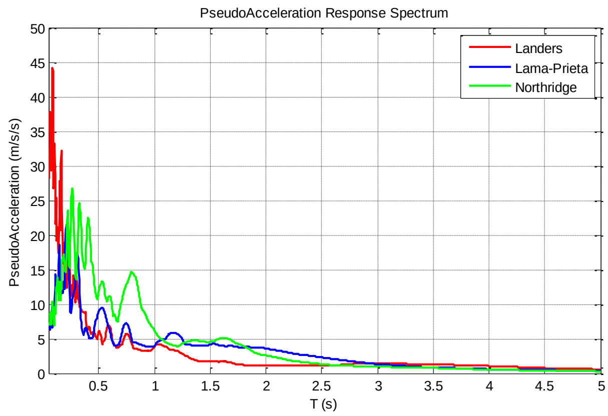


Fig. 10 Pseudo-acceleration response spectrum for 0.2 damping ratios

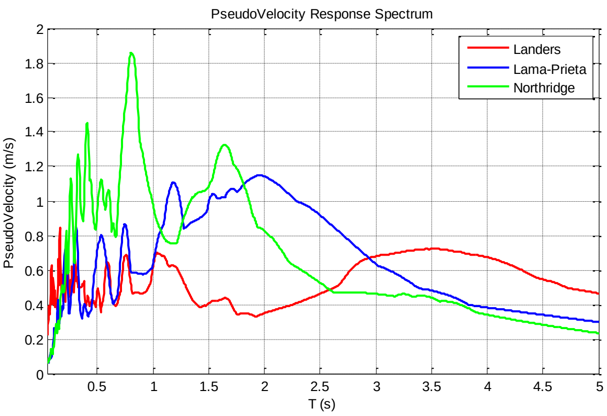


Fig. 11 Pseudo-velocity response spectrum for 0.2 damping ratios

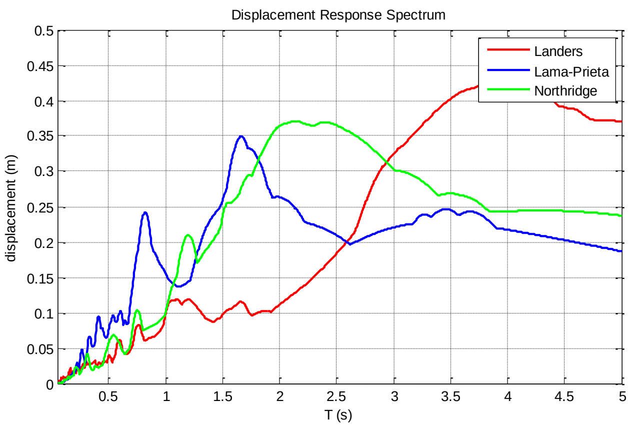


Fig. 12 Displacement response spectrum for 0.2 damping ratios

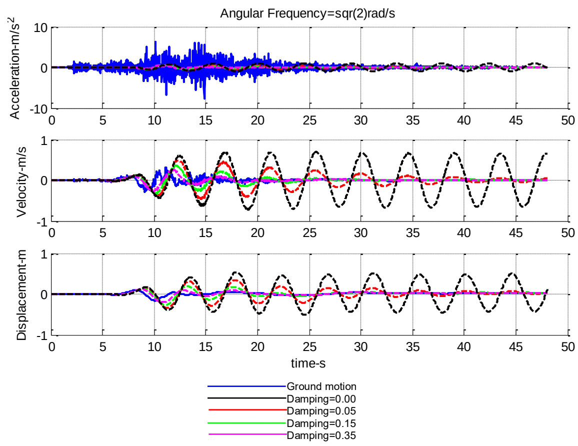


Fig. 13 The isolator response to the Landers earthquake for different damping ratios

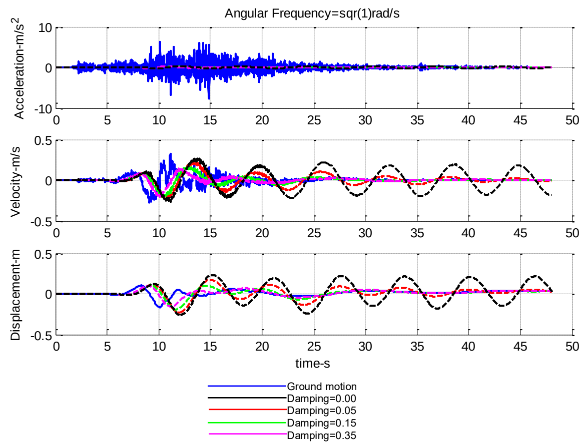


Fig. 14 The isolator response to the Landers earthquake for different damping ratios

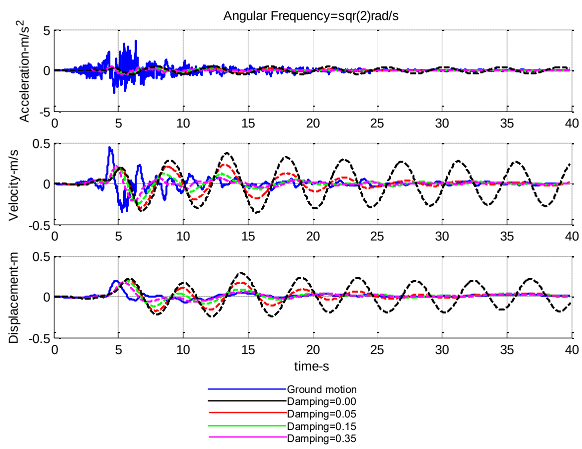


Fig. 15 The isolator response to the Loma Prieta earthquake for different damping ratios

3. The isolator response to some earthquakes

By considering $g/l = 2$ and 1 with damping ratios of 5, 15, and 35%, results are given. The isolator responses were plotted in Figs. 10 to 18.

When g/l is equal to 2 for a structure, then by considering the effective gravity (for example, $g_{eff} = g/2$), g/l reduces from 2 to 1.

Stations used for records are as follows:

a. The Landers (USA) earthquake of June 28, 1992. Recording station: 000 SCE STATION 24, Source: PEER Strong Motion Database

b. The Loma Prieta (USA) earthquake of October 18, 1989. Recording station: 090 CDMG STATION 47381,

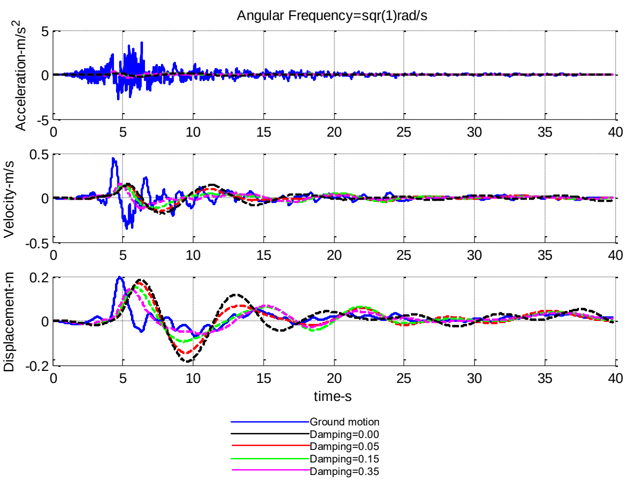


Fig. 16 The isolator response to the Loma Prieta earthquake for different damping ratios

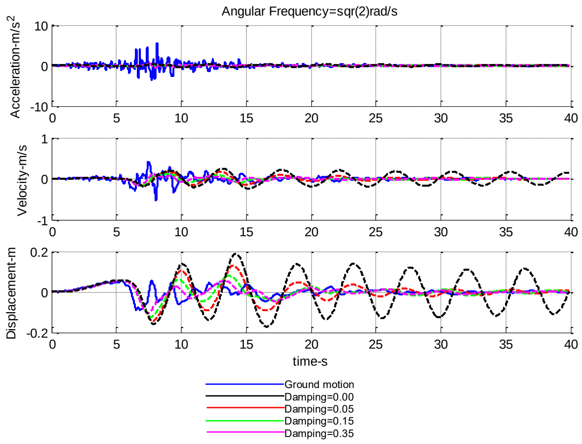


Fig. 17 The isolator response to the Northridge earthquake for different damping ratios

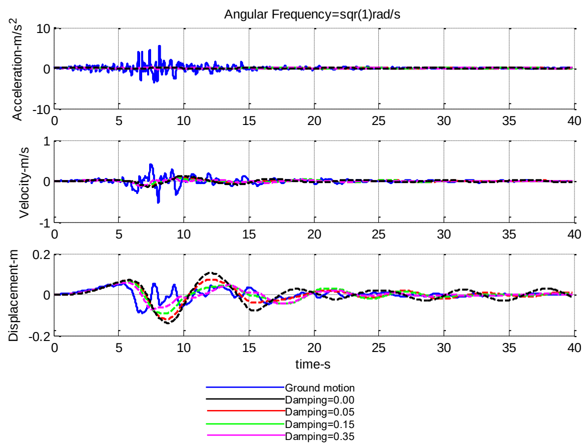


Fig. 18 The isolator response to the Northridge earthquake for different damping ratios

Source: PEER Strong Motion Database
c. The Northridge (USA) earthquake of January 17, 1994. Recording station: 090 CDMG STATION 24278, Source: PEER Strong Motion Database

The aim of designing structures is to ensure proper performance against external factors, like earthquakes. Seismic isolation is a method to increase the safety of structures against earthquakes by changing the received vibration into a non-destructive form. For this purpose, it is necessary to analyze the vibration created in the isolated structure. The findings indicate that there has been a significant decrease in the response acceleration of the structure that has been isolated. This is because of the difference between the natural period of the isolator and the earthquake's dominant period. The displacement and velocity diagrams further demonstrate how the structure moves smoothly during an earthquake, eliminating any jarring movements that could pose a danger to the occupants and ensuring their safety.

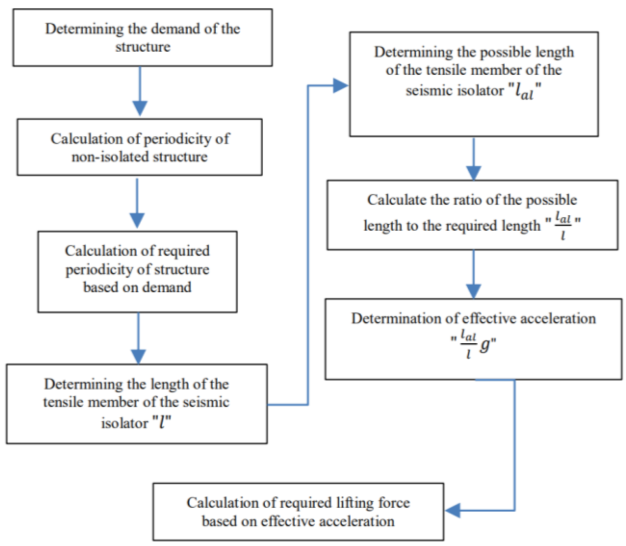


Fig. 19 A simplified design chart

4. Design chart

To design a chart proposition for base isolation, several parameters are needed, but as a primarily suggested suggestion, a simplified chart can be as shown in Fig. 19.

5. Experimental tests

An experimental model was built and tested at Tabriz University. The model had four legs. Each leg was a pendulum column. Two plates were used instead of the ceiling and floor. In order to achieve effective gravity, a spring rod was placed (Fig. 20(a)). The model was mounted on a shaking table and tested. Fig. 20(b) shows a frame of the video taken in the lab. As it is seen from Fig. 20(b), the piers are faded because of the intensity of movement, while the upper part remains stationary.

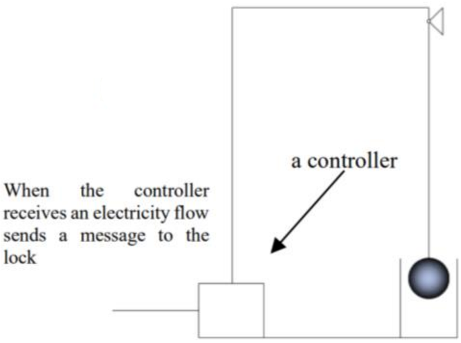
To address the issue of low stiffness in the system, a simple electrical setup is employed to stabilize it against undesirable vibrations.

This setup resembles the proposed case depicted in Fig. 21(a). In Fig. 21(b), a pendulum resides within a metallic container and is integrated into an electrical circuit,

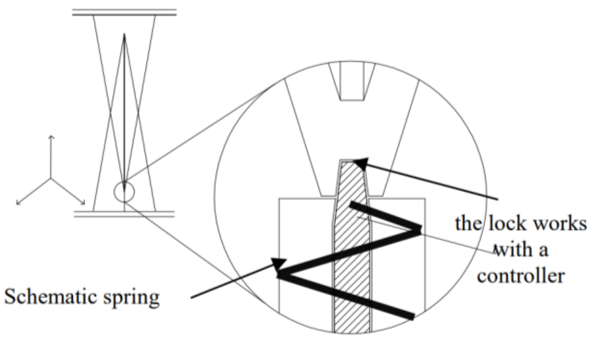
(a)

(b)

Fig. 20 (a) Pendulum column with a rod spring and (b) The system under test (Azizi and Barghian 2023b)



(a)



(b)

Fig. 21 A schematic suggests lock system to control system stability (Azizi and Barghian 2023b)

effectively serving as a switch for the circuit. Under normal conditions, when there is no significant vibration, there is no contact between the pendulum and the container. However, when vibrations surpass a predetermined threshold, the pendulum comes into contact with the container, thereby triggering a signal to a controller, which subsequently releases a lock.

6. Conclusions

An isolator system was studied, which was called the “pendulum column” under “effective gravity”. The governing equation of this isolator was derived. A program was written in MATLAB to solve the driven equations. Three different earthquake records were chosen, and then three different damping ratios were used. The displacement and acceleration responses of the isolator were plotted. In the plots $\frac{g}{l}$ was the effective calculation criterion. Here, two $\frac{g}{l}$ amounts of 2 and 1 were considered. The diagrams showed that the studied isolator had reduced the acceleration very effectively. Among the different damping ratios, those with ratios greater than 15% had better results. “Effective gravity” played a good role in the response of the system by increasing the system’s natural period. Finally, the tested model verified the theory.

References

Auad, G., Castaldo, P. and Almazán, J.L. (2022), “Seismic reliability of structures equipped with LIR-DCFP bearings in terms of superstructure ductility and isolator displacement”, *Earthq. Eng. Struct. Dyn.*, **51**(13), 3171-3214. https://doi.org/10.1002/eqe.3719.

Azizi, A. and Barghian, M. (2015), “Using pendulum column as base isolation”, M.Sc. dissertation, University of Tabriz, Tabriz, Iran.

Azizi, A. and Barghian, M. (2023a), “Introducing a multi-layer pendulum isolator and investigating its effect on structures’ responses during some earthquakes”, *Struct.*, **57**, 105206. https://doi.org/10.1016/j.istruc.2023.105206.

Azizi, A. and Barghian, M. (2023b), “Investigating the “pendulum column” isolator with flexible piers”, *Earthq. Struct.*, **24**(6), 405-413. https://doi.org/10.12989/eas.2023.24.6.405.

Barghian, M. and Shahabi, A.B. (2007), “A new approach to pendulum base isolation”, *Struct. Control Health Monit.*, **14**, 177-185. https://doi.org/10.1002/stc.115.

Cancellara, D., Angelis, F.D. and Pasquino, M. (2013), “A novel seismic base isolation system consisting of a lead rubber bearing in series with a friction slider”, *J. Appl. Mech. Mater.*, **256**, 2174-2184. https://doi.org/10.4028/www.scientific.net/AMM.256-259.2174.

Cao, L. and Li, C. (2021), “A high performance hybrid passive base-isolated system”, *Struct. Control Health Monit.*, **29**, (3), 1-26, e2887. https://doi.org/10.1002/stc.2887.

Castaldo, P., Amendola, G., Giordano, L. and Miceli, E. (2022), “Seismic reliability assessment of isolated multi-span continuous deck bridges”, *Ing. Sismica*, **39**(3), 26-51.

Castaldo, P. and Miceli, E. (2023), “Optimal single concave sliding device properties for isolated multi-span continuous deck bridges depending on the ground motion characteristics”, *Soil Dyn. Earthq. Eng.*, **173**, 108128. https://doi.org/10.1016/j.soildyn.2023.108128.

Chen, X., De Domenico, D. and LI, C. (2023), “Seismic resilient design of rocking tall bridge piers using inerter-based systems”, *Eng. Struct.*, **281**, 115819. https://doi.org/10.1016/j.engstruct.2023.115819.

Cirelli, M., Gregori, J., Valentini, P.P. and Pennestrí, E. (2019),

"A design chart approach for the tuning of parallel and trapezoidal bifilar centrifugal pendulum", *Mech. Mach. Theory*, **140**, 711-729. https://doi.org/10.1016/j.mechmachtheory.2019.06.030.
Deringöl, A.H. and Güneyisi, E.M. (2019), "Effect of friction pendulum bearing properties on behaviour of buildings subjected to seismic loads", *Soil Dyn. Earthq. Eng.*, **125**, 105746. https://doi.org/10.1016/j.soildyn.2019.105746.
Gino, D., Miceli, E. and Castaldo, P. (2023), "Seismic reliability analysis of isolated deck bridges using friction pendulum devices", *Proc. Struct. Integr.*, **44**, 1435-1442. https://doi.org/10.1016/j.prostr.2023.01.184.
Islam, S., Jumaat, M.J. and Zamin, M. (2011), "Seismic isolation in buildings to be a practical reality: Behavior of structure and installation technique", *J. Eng. Technol. Res.*, **3**(4), 99-117.
Izumi, M. (1988), "State-of-the-art report: Base isolation and passive seismic response control", *Proceedings of the 9th World Conference on Earthquake Engineering*, Tokyo, Japan, August.
Kim, Y., Shahriyer, H. and Hu, J. (2022), "Seismic performance evaluation according to HSS and CFST columns of 3D frame buildings with rubber friction bearing (RFB)", *Mater.*, **15**(4), 1281. https://doi.org/10.3390/ma15041281.
Kelly, J.M. (1986), "A seismic base isolation: Review and bibliography", *Soil Dyn. Earthq. Eng.*, **5**(4), 202-216. https://doi.org/10.1016/0267-7261(86)90006-0.
Li, C., Chang, K., Cao, L. and Huang, Y. (2021), "Performance of a nonlinear hybrid base isolation system under the ground motions", *Soil Dyn. and Earthq. Eng.*, **143**, 106589. https://doi.org/10.1016/j.soildyn.2021.106589.
Li, Y. and Li, J. (2014), "Base isolator with variable stiffness and damping: Design, experimental testing and modelling", *Proceedings of the 23rd Australasian Conference on the Mechanics of Structures and Materials*, Byron Bay, Australia, December.
Lin, A.N. and Shenton, III H.W. (1992), "Seismic performance of fixed base and base isolated steel frames", *ASCE, J. Eng. Mech.*, **118**(5), 921-941. https://doi.org/10.1061/(ASCE)0733-9399(1992)118:5(921).
Luco, J.E. (2014), "Effects of soil-structure interaction on seismic base isolation", *Soil Dyn. Earthq. Eng.*, **66**, 167-177. https://doi.org/10.1016/j.soildyn.2014.05.007.
Lupășteanu, V., Soveja, L., Lupășteanu, R. and Chingălată, C. (2019), "Installation of a base isolation system made of friction pendulum sliding isolators in a historic masonry orthodox church", *Eng. Struct.*, **188**, 369-381. https://doi.org/10.1016/j.engstruct.2019.03.040.
Monfared, H., Shirvani, A. and Nwaubani, S. (2013), "An investigation into the seismic base isolation from practical perspective", *Int. J. Civil Struct. Eng.*, **3**(3), 451-463.
Naeim, F. and Kelly, J.M. (1999), *Design of Seismic Isolated Structures from Theory to Practice*, John Wiley & Sons, Hoboken, NJ, USA.
Shah, V.M. and Soni, D.P. (2017), "Response of the double concave friction pendulum system under triaxial ground excitations", *Proc. Eng.*, **173**, 1870-1877. https://doi.org/10.1016/j.proeng.2016.12.240.
Shenton, III H.W. and Lin, A.N. (1993), "Relative performance of fixed based and base isolated concrete frame", *ASCE Struct. Eng.*, **119**(10), 2952-2968. https://doi.org/10.1061/(ASCE)0733-9445(1993)119:10(2952).
Shrimali, M.K., Bharti, S.D. and Dumne, S.M. (2015), "Seismic response analysis of coupled building involving MR damper and elastomeric base isolation", *Ain Shams Eng. J.*, **6**, 457-470. https://doi.org/10.1016/j.asej.2014.12.007.
Su, L., Ahmadi, G. and Tadjbakhsh, I.G. (1991), "Performance of sliding resilient-friction base-isolation system", *ASCE J. Struct. Eng.*, **117**(1), 165-181. https://doi.org/10.1061/(ASCE)0733-9445(1991)117:1(165).
Thomas, T. and Mathai, A. (2016), "Study of base isolation using friction pendulum bearing system", *J. Mech. Civil Eng.*, **2006**, 19-23.
Vrochidou, E., Alvanitopoulos, P. F., Andreadis, I., Elenas, A. and Mallousi, K. (2014), "Synthesis of artificial spectrum-compatible seismic accelerograms", *Measure. Sci. Tech.*, **25**(8), 085002.
Wright, K.R. and Zegarra, A.V. (2000), "*Machu Picchu: A Civil Engineering Marvel*", ASCE Press, Reston, VA, USA.
Xiuting, S., Jian, X. Feng, W. and Shu, Z. (2018), "A novel isolation structure with flexible joints for impact and ultralow-frequency excitations", *Int. J. Mech. Sci.*, **146**, 366-367. https://doi.org/10.1016/j.ijmecsci.2018.08.009.
Zhong, C. and Christopoulos, C. (2022), "Shear-controlling rocking-isolation podium system for enhanced resilience of high-rise buildings", *Earthq. Eng. Struct. Dyn.*, **51**(6), 1363-1382. https://doi.org/10.1002/eqe.3619.

CC

Nomenclature

e	earth
E_{kin}	the kinetic energy
E_{spr}	the energy of the spring
E_{pot}	the potential energy
g	acceleration of gravity
g_{eff}	the effective gravity
h	the height of the pendulum from its equilibrium state
k	the stiffness of the spring
l	the length
m	the mass
N	the uplifting force under the pendulum
s	the structure mass
T	the pendulum period
v_s	the velocity of the pendulum
W	weight=mg
w_{ext}	the external work
x_e	the horizontally displaced amount of earth
x_s	the horizontally displaced amount of the lower part of the pendulum
$\dot{x}_s$	the velocity of x_s
$\ddot{x}_s$	the acceleration of x_s
Δ	the initial compressed length of the spring that is due to the creation of an uplifting force
θ	the angle of the pendulum rod after rotating